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THE MECHANIC'S GUIDE.

THE

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A Practical Hand-Book

FOR THE USE OF

ENGINEERS, MECHANICS, ARTIZANS, &c.,

COMPRISING

ARITHMETIC, GEOMETRY, MENSURATION, VELOCITIES, WHEEL-GEARING, SCREW-CUTTING, THE STEAM-ENGINE, &c., AND THE GENERAL PRINCIPLES OF MECHANISM; WITH A NUMBER OF USEFUL RECIPES AND COPIOUS TABLES FOR PRACTICAL USE.

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THE MECHANIC'S GUIDE.

PREFACE.

In offering the following pages to the consideration of the Mechanical Skill and Intelligence of the Working Men of England and other countries, the Author is desirous of pointing out in a few words the object of their publication.

It is simply this: the gathering together into one connected *whole* the principal subjects relating to various branches of the Mechanical Art, and placing before readers who may not have much leisure for study a concise and simple explanation of General Principles, together with Illustrations of their adaptation to practical purposes.

While fully sensible of the imperfect way in which this aim has been carried out, he yet trusts that the book, such as it is, may answer to its title, and fill to some extent, at least, the existing want for a work of this kind. Knowing well the special difficulties of *Self-Teachers*—of those who are obliged to prosecute study without direct assistance of any kind—he has endeavoured throughout to keep the requirements of such distinctly in view, giving occasionally the results of his own experience. No methods of working or calculating different systems are recommended but the most complete and universally acknowledged.

In conclusion, the Author can only say that the book is the work of a Practical Mechanic, who may not have the language of a Professor at command, but who has tried, to the best of his ability, to supply, honestly and thoroughly, information such as he knows to be greatly needed by intelligent mechanics at the present time.

WM. V. SHELTON.

38 RUE HENDECK BASHI,
CONSTANTINOPLE.

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Inscribed

TO

SIR PHILIP FRANCIS,

JUDGE, AND CONSUL-GENERAL OF HER BRITANNIC MAJESTY,
AT CONSTANTINOPLE;

AS

A TRIBUTE TO HIS INDEFATIGABLE ZEAL
IN THE INTERESTS AND WELFARE OF THE MECHANICS EMPLOYED
THROUGHOUT THE DOMINIONS

OF

His Majesty the Sultan of Turkey,

AND TO HIS EARNEST ENDEAVOURS TO ADVANCE THE INFLUENCE
OF MECHANICAL

SKILL AND LABOUR.

g is the common expression for *Gravity*, as $=32{\cdot}166$, $2g=64{\cdot}33$, $\sqrt{2g}=8{\cdot}02$ feet.

^{2}, ^{3}, ^{4}, *added* or set *superior* to a symbol signify that the number, &c., is to be squared, cubed, &c.; thus 4^2, means that 4 is to be multiplied by 4; 4^3, that it is to be cubed, as 4^3 is $=4\times4\times4=64$. The power or number of times a number is to be multiplied by itself is shewn by the number added, as ^{2}, ^{3}, ^{4}, ^{5}, &c.

$\frac{1}{2}$, $\frac{1}{3}$, &c., set *superior* to a number signify the square or cube root of that number.

$\frac{4}{2}$, $\frac{4}{3}$, $\frac{6}{3}$, &c., set *superior* to a number signify the square or cube root of the 4th power, &c.

$^{1\cdot7}$, $^{3\cdot6}$, &c., set *superior* to a number signify the *tenth* root of the 17th power, &c.

$'$ $''$ set *superior* to a figure or figures signify *feet* and *inches*.

Simple Addition, Subtraction, Multiplication, Division, and Vulgar Fractions.

ADDITION.

When any numbers are taken together or added, the resulting number is called their *sum*.

Example.

Add	94163
	21954
	7812
	593
	35647
	4895
	165064—*Result.*

In order to add whole numbers together, we first place them *under one another*, with their units-figures in the same vertical line; we then add these figures, thus:—

5 and 7 are 12, and 3 are 15, and 2 are 17, and 4 are 21, and 3 are 24—*i.e*, 24 units, or 2 tens and 4 units. We set the 4 under the units-figures, to be the units-figure of the result, and carry the 2 tens to be added to the second column; adding this in the same manner, beginning with the 2 carried, thus—2 and 9 are 11, and 4 are 15, &c., we find the sum of the column to be 36—*i.e.*, 36 tens, or 3 tens of tens (*i.e.*, 3 hundreds) and 6 tens; we set the 6 under the *tens-figures, to be the tens-figure* of the result, and carry the 3

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hundreds to the third or hundreds column: pursuing the same course with this, we find the sum of this column to be 40—*i.e*, 40 hundreds or 4 tens of hundreds (*i.e.*, 4 thousands) and 0 hundreds; we set the 0 under the hundreds-figures to be the hundreds-figure of the result, and carry the 4 thousands to the next or fourth column, &c.

Notice.—Any sum may be set at pleasure in Addition, and the answers proved by repeating the operation, beginning with the top figures of the units-column, when the result will be the same if the sum be worked correctly.

Examples in Addition.

(1.)	(2.)	(3.)	(4.)
321413	543123	536123	123456
452734	234512	453215	234561
130421	713145	1234	345612
3718	104234	4231	456123
24561	36142	51234	561234
341323	3451	613254	612345

(5.)	(6.)	(7.)	(8.)
761284	657890	692387	768453
612874	278679	4956	358428
8719	5798	87958	8796
46759	67843	769378	54937
587999	489567	5790	495
987678	37429	87658	876578

Answers to the Eight Examples.

(1.) 1274170	(2.) 1634607	(3.) 1659291	(4.) 2333331
(5.) 3005313	(6.) 1537206	(7.) 1648127	(8.) 2067687

Example 9.

Add together five hundred and ninety-seven thousand six hundred and eighty-five, forty-nine thousand three hundred and seven, four hundred and nine thousand and sixty-seven, fourteen thousand and nineteen, seven hundred thousand and seventy-four, sixty-five thousand and nine.

Example 10.

Add together seven hundred and seven thousand four hundred *and fifty-nine, ninety-eight* thousand and seventy-four, six thousand *eight hundred and seven,* five hundred thousand three hundred and

nine, seven thousand nine hundred and seventy-eight, nine hundred and nine thousand nine hundred and ninety-nine.

Example 11.

Add together fifty-five millions seven hundred thousand and five, seven hundred millions nine hundred and eight thousand two hundred and five, seventy-six millions fourteen thousand and fifty-nine, eight hundred and seventy-seven millions nine hundred and two thousand and forty-seven, seven millions eight hundred and four thousand five hundred and twelve, five hundred and seventy-five millions eight hundred and one thousand and ninety-nine.

Example 12.

Add together three hundred and nine millions four hundred and seventeen thousand and eighty-seven, six hundred and seventy-five thousand and forty-nine, seven thousand and ninety-seven millions eight hundred and fourteen thousand three hundred and five, seventy-nine millions five hundred and four thousand and forty-nine, six thousand and seventy-eight millions four hundred and thirty-nine thousand six hundred and forty-seven, seven thousand millions eight hundred and seventy-six thousand four hundred and twenty-nine.

SUBTRACTION.

When one number is taken from another, or subtracted, the result is called the remainder, or the difference.

Example.

From	794327
Take	342814
	451513—*Difference.*

In order to subtract one whole number from another, we first place the number to be subtracted under the other, with their units-figures in the same vertical line; we then take the units-figure 4 of the lower number from that of the other, 7, thus—4 from 7, 3, and we place the 3 under the units-figures, to be the units-figure of the result; we then proceed to the tens-figures, and say, 1 from 2, 1, and we set *down 1* under the tens-figures; then to the *hundreds-figures, and say 8 from 3*; but as this cannot be done, we

take or borrow 1 from the 4 thousands (leaving 3 thousands), and treat it as 1 ten of hundreds, we shall now have 13 hundreds in the upper line; we can now say 8 from 13, 5, and we set down 5 as the hundreds-figure of the result; and we have now to take 2 thousands from 3 thousands, or which is just the same, but more convenient in practice, instead of supposing the upper figure, 4, *diminished* when we borrow 1, we may suppose the lower corresponding figure 2 *increased*—*i.e.*, we may carry one to it, and say 3 from 4, and so through the entire calculation.

Notice.—Any question may be set at pleasure in subtraction, and the answer *proved* by *adding* the remainder to the *lower* number, when the result will be the *upper line* if the question be worked correctly.

EXAMPLES IN SUBTRACTION.

(1.)	(2.)	(3.)	(4.)
765439	697438	758452	543625
343418	635036	418234	492708
422021	62402	340218	50917

(5.)	(6.)	(7.)	(8.)
683125	712345	564307	702306
492816	538159	479176	475429
190309	174186	85131	226877

EXAMPLE 9.

From six hundred and nine thousand seven hundred and one, take three hundred and ninety-seven thousand and forty-nine. *Ans.* 212652.

EXAMPLE 10.

From four hundred and fifty thousand and ninety-four, take ninety-nine thousand nine hundred and nine. *Ans.* 350185.

EXAMPLE 11.

From seven hundred and eighteen millions fourteen thousand and fifty-seven, take ninety-seven millions eight hundred and four thousand seven hundred and sixteen. *Ans.* 620209341.

EXAMPLE 12.

From fifty-three thousand millions eighteen thousand and ninety-seven, take forty thousand five hundred and twenty-eight millions *seven hundred* and six thousand seven hundred and nine.

Ans. 12471311388.

MULTIPLICATION.

Multiplication is the method of finding what number would result from adding several of the *same* numbers together; thus, if we add 6 sevens together, the result is $7+7+7+7+7+7=42$, being the same number as that given in the multiplication-table for the value of 6 times 7; and since the same is also the sum of 7 sixes, or the value of 7 times 6, it follows that when *two* numbers are multiplied together, it matters not which we take as the multiplier.

The numbers multiplied in any case are called *factors*, and the result is called the *product*.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

EXAMPLE 1.

$$\begin{array}{r} 3467 \\ \times\ \ 2 \\ \hline 6934 \end{array}$$

When the multiplier, as in Example 1, is not higher than 12, we first set it with the units-figure under that of the *multiplicand* (or number multiplied). Then we begin to multiply as follows:—Twice 7 are 14, 4 and carry 1—*i.e.*, we set down the 4 units under the units-figures, and carry the 1, which means 1 ten, to be added to the tens. We now proceed: twice 6 are 12—*i.e.*, 12 tens, since 6 means 6 tens, and 1 (*i.e.*, the 1 carried) is 13, 3 and carry 1—*i.e*, we set down the 3 tens and carry the 1, which means 1 ten of tens, or 1 hundred, to be added to the hundreds, and so on throughout the line.

EXAMPLE 2.

3467.....	2	= remainder of $(3+4+6+7)\div 9$
692.....	8	= remainder of $(6+9+2)\div 9$
6934		
31203		
20802		
2399164—*Result*...	7	= remainder of $(2+$ &c., $+4)\div 9$

When the multiplier, as in Example 2, is higher than 12, we first set it under the multiplicand as before; and having multiplied the upper line by the units-figure, 2, of the lower, as in Example 1, we now multiply by the tens-figure, 9, saying 9 times 7 are 63 (*i.e.*, 63 tens, since 9 means 9 tens), 3 and carry 6—*i.e.*, we set down the 3 tens, and carry the 6 tens of tens or hundreds-figure. We proceed in the same manner with the third multiplier, 6, and then add up the separate lines, when the result is the product required. The figures on the right—2, 8, and 7—will be explained presently.

EXAMPLE 3.

37218
6

223308
7

1563156—*Result.*

Since it is immaterial which number we take as a multiplier, it is best always to choose that which is the *simplest*, and if it can be separated into two or more factors each less than 12 (thus, $42 = 6\times 7$), *we may multiply* separately by each, as in Example 3.

Notice.—Any number which can be separated into factors is called a *composite number;* any number which cannot be so separated, such as 7, 11, 13, 17, &c., is called a *prime number.*

EXAMPLE 4.

3241.....	1
2700.....	0
2268700	
6482	
8750700—*Result*.....	0

If the multiplier ends with one or more cyphers, the question may be worked as in the above example, by which method many useless cyphers are dispensed with.

Notice.—Any question may be set at pleasure in multiplication, and the answers *proved* either by *repeating* the operation with the original multiplicand for multiplier, or by the process of *casting out nines,* as follows:—Add up the figures in the multiplicand, divide this by 9, and set down the remainder; do the same with the multiplier, then do the same with the product of these *remainders,* and with the product of the two numbers; and if the *new* remainders are the same, the product is most probably right, but if different, it is certainly wrong. Thus, in Example 2, the *first pair of remainders* are 2 and 8, and their product 16; the remainder from this is 7, the same as from the answer. In Example 4 the first pair of remainders are 1 and 0, and their product 0; the remainder from this is 0, the same as from the answer.

It is desirable that the reader should apply one or both of these methods to the examples given below.

EXAMPLES IN MULTIPLICATION.

(1.)	345673	× 2	(2.)	457632	× 3
(3.)	415763	× 4	(4.)	371281	× 5
(5.)	635432	× 6	(6.)	421675	× 7
(7.)	378914	× 8	(8.)	476539	× 8
(9.)	435976	× 9	(10.)	978564	× 11
(11.)	496782	× 12	(12.)	876549	× 12
(13.)	378125	× 16	(14.)	456932	× 18
(15.)	712436	× 24	(16.)	543817	× 27
(17.)	593654	× 30	(18.)	697128	× 36
(19.)	765438	× 40	(20.)	596437	× 45

(21.)	642198	× 60	(22.)	756328	× 72
(23.)	814765	× 84	(24.)	913748	× 96
(25.)	234915	× 123	(26.)	704745	× 615
(27.)	469830	× 369	(28.)	391525	× 861
(29.)	1174575	× 2214	(30.)	3523725	× 2583
(31.)	154366	× 7749	(32.)	231549	× 8856
(33.)	463098	× 7380	(34.)	1389294	× 8900
(35.)	926196	× 7896	(36.)	2778588	× 9867

ANSWERS TO THE ABOVE EXAMPLES FROM 1 TO 12.

(1.)	691346	(2.)	1372896	(3.)	1663052
(4.)	1856405	(5.)	3812592	(6.)	2951725
(7.)	3031312	(8.)	3812312	(9.)	3923784
(10.)	10764204	(11.)	5961384	(12.)	10518588

ANSWERS IN MULTIPLICATION FROM 13 TO 36.

(13.) 378125 × 16
2268750 = *Product* × 6

6050000—*Answer.*

(14.) 456932 × 18
3655456 = *Product* × 8

8224776 —*Answer.*

(15.) 712436 × 24
12

8549232
2

17098464—*Answer.*

(16.) 543817 × 27
9

4894353
3

14683059—*Answer.*

(17.) 593654 × 30
30

17809620—*Answer.*

(18.) 697128 × 36
6

4182768
6

25096608—*Answer.*

(19.) 765438 × 40
40

30617520—*Answer.*

(20.) 596437 × 45
9

5367933
5

26839665—Answer.

(21.) 642198 × 60
60

38531880—*Answer.*

(22.) 756328 × 72
9

6806952
8

54455616—*Answer.*

(23.) 814765 × 84
12

9777180
7

68440260—*Answer.*

(24.) 913748 × 96
12

10964976
8

87719808—*Answer.*

(24.) (Otherwise) 91374800
(100 – 4)
(Subtract) 3654992 =
Pro. by 4

87719808—
Answer.

(25.) 234915
123

704745
2818980

28894545—*Answer.*

(26.) 704745
615

3523725
704745
4228470

433418175—*Answer.*

(27.) 469830
369

4228470 × 40 times
169138800 = 360 „

173367270—*Answer.*

(28.) 391525
861

391525
2349150
3132200

337103025—*Answer.*

(29.) 1174575
2214*

4698300
1174575
2349150
2349150

2600509050—*Answer.*

All marked thus * *see* "Remarks," page 12.

(30.) 3523725 × 2583

10571175
28189800
17618625
7047450

9101781675—*Answer.*

(31.) 154366 × 7749*

1389294
617464
1080562
1080562

1196182134—*Answer.*

(32.) 231549 × 8856*

1389294
1157745
1852392
1852392

2050597944—*Answer.*

(33.) 463098 × 7380

37047840
1389294
3241686

3417663240—*Answer.*

(34.) 1389294 × 8900

1250364600
11114352

12364716600—*Ans.*

(35.) 926196 × 7896*

5557176
8335764
7409568
6483372

7313243616—*Ans.*

(36.) 2778588 × 9867

19450116
16671528
22228704
25007292

27416327796—*Ans.*

Remarks.

Abridgments similar to that in Example 27 may be used in Examples 29, 31, 32 and 35; thus, in Example 29: We may take first 2 thousand times, then 2 hundred times, 2×7 times, as here shewn; and an exactly similar form may be used in Examples 31 and 32. In Example 30 the multiplier is $= 2400 + 180 + 3$, and therefore allows us to take 3, 60 times 3, and 800 times 3. In Example 35 we may proceed from left to right, taking 7, 8, and *12 times 8.*

1174575 × 2214

2349150
2349150 × 7
16444050

2600509050

DIVISION.

Division is the method of finding how often *one* number is contained in *another—i.e.*, how often one number *must be taken* to *make up* another. Hence Division bears the same relation to Subtraction as Multiplication bears to Addition; for we might go on subtracting the *divisor*, or number by which the division is made, from the *dividend* or number divided, and then from the first remainder, and then from the second remainder, and so on, until the final remainder is either zero, or is *less* than the divisor itself; and if we counted the number of times we had subtracted it, this would be the result required, or as it is properly called, the *quotient*. But the Multiplication Table will enable us much more easily to divide one number by another; thus—7 times 9 are 63, and if we divide 63 by 7 we shall have the quotient 9, or if by 9 the quotient 7; and the method of applying it to more difficult cases will be seen by what follows.

EXAMPLE 1.

4)2379

$594\frac{3}{4}$—*Result.*

When the divisor, as in Example 1, is not higher than 12, we first set it in a loop before the dividend, then we take the first figure of the dividend, 2—*i.e.*, 2 thousands, but since 4 will not be contained at all in this, we take then the first two figures, 23—*i.e.*, 23 hundreds, and say, 4 is contained in 23, 5 times and 3 over, we set down 5—*i.e.*, 5 hundreds, in the quotient, and carry the 3 hundreds or 30 tens to the tens-figure, 7, of the dividend. We have now 37 tens to be divided by 4; we say, therefore, 4 is contained in 37, 9 times and 1 over, we set down the 9—*i.e.*, 9 tens, in the quotient, and carry the 1 ten or 10 units to the units-figure, 9, of the dividend. We now have 19 units to be divided by 4; we say, therefore, 4 is contained in 19, 4 times and 3 over, we set down the 4—*i.e.*, 4 units, in the quotient, and place, as is usual, the final remainder 3 over the divisor, with a line between them, thus, $\frac{3}{4}$ (three-fourths), a quantity meaning $3 \div 4$, and called a *fraction*, of which more will be said hereafter.

It appears, then, that 4 will be contained 594 times in 2379 with 3 over—*i.e.*, we *might* subtract 4 from 2379 594 times, and *still have 3 remaining*. *This is* an example in *Short* Division.

EXAMPLE 2.

42)379543(9036$\frac{31}{42}$—*Quotient.*
378

154
126

283
252

31—*Remainder.*

When the divisor, as in Example 2, is higher than 12, we place it as before in a loop before the dividend, and the quotient in a loop after it. We find that 42 will not be contained in the 3 (*i.e.*, 3 hundreds of thousands), nor in the 37 (*i.e.*, 37 tens of thousands), but will be contained 9 times in the 379 (*i.e.*, 379 thousands); or, which is the same thing, but more convenient in practice, we take the first figure only of the dividend, and say 4 is contained in 37, 9 times; we set, therefore, the 9 (*i.e.*, 9 thousands) in the quotient, and multiplying 42 by 9, subtract the product, 378 (*i.e.*, 378 thousands), from the dividend, and we have now the remainder—*i.e.*, 1 thousand or 10 hundreds, to be carried to the hundreds. We then take the hundreds-figure, 5, of the dividend, and have now 15 hundreds to be divided by 42; but as we cannot divide 15 by 42, we set 0 (*i.e.*, 0 hundreds) in the hundreds place of the quotient, and have now 15 hundreds or 150 tens to be carried to the tens; we then take the tens-figure, 4, of the dividend, and have now 154 tens to be divided by 42; we then find that 4 is contained in 15, 3 times, and we set the 3, *i.e.*, 3 tens, in the quotient, and so on till at length we have the final remainder, 31, which we set over the divisor as a fraction, and have the whole quotient, 9036$\frac{31}{42}$. This is an example in Long Division.

EXAMPLE 3.

But when the divisor, as in this case, is made up of two or more factors less than 12, it is often more convenient to divide by each separately, as follows :—

6)379543

7)63257$\frac{1}{6}$

9036$\frac{31}{42}$—*Result.*

There is here a fraction $\frac{1}{6}$ over in the first quotient, and a remainder $\frac{5}{6}$ in the second, which, according to our previous practice, should be written $\frac{5\frac{1}{6}}{7}$; but such an expression may always be simplified, as will be shewn hereafter, by putting the remainder, $5\frac{1}{6}$, in the form $\frac{31}{6}$, which we obtain by multiplying the 5 by the 6 and adding in the 1, and then multiplying the 6 by the 7, so making $\frac{31}{42}$, the same as the fraction obtained by the other method.

EXAMPLE 4.

$$39\,00)7134\,53(182\tfrac{3600}{5300}\tfrac{53}{00}$$

39

323
312

114
78

3653

In this example, and in all others where there are cyphers at the end of the divisor, the work may be abridged by marking off with a comma or point these cyphers, and as many figures also from the right of the dividend; then we proceed, 3 is contained in 7 twice; but on trial we should find that 2 would be too great a number for the first figure in the quotient (which arises by using for the divisor the figure 3 instead of 39; this difficulty will sometimes occur, but not so often as to embarrass the student when he gets accustomed to division). We set, therefore, 1 as the first figure in the quotient, and go on as before, until we have taken down all the figures before the point in the dividend, and then we complete the last remainder by taking down the two figures cut off, and place them over the divisor as a fraction.

Notice.—Any question may be set at pleasure in Division, and the answers *proved* by either of the methods given in Multiplication, since the *product* of the *divisor* and *quotient* (if the question be worked correctly) will give the *dividend*, diminished, however, by the remainder or *upper number* of the fraction, if any. Thus, in Example 2, the divisor is 42 and the quotient 9036, and dividing by 9, the remainders from these are 6 and 0; the product of these is 0, and the dividend, diminished by the remainder, 31, is 379512, and the remainder from this is also 0. In Example 4, the divisor is 3900 *and the quotient 182,* the remainders being 3 and 2; the

product of these is 6, and the dividend, diminished by the remainder, 3653, is 709800, and the remainder from this is also 6. The reader should apply one or other of these methods of proof in the following examples :—

Examples in Division.

(1.)	432516	÷2	(2.)	351789	÷3
(3.)	543756	÷4	(4.)	713915	÷5
(5.)	385734	÷6	(6.)	516824	÷7
(7.)	465328	÷8	(8.)	395424	÷8
(9.)	567035	÷9	(10.)	457848	÷11
(11.)	716855	÷12	(12.)	936571	÷12
(13.)	2366745	÷15	(14.)	7954326	÷18
(15.)	6342576	÷24	(16.)	6549372	÷36
(17.)	4733491	÷45	(18.)	5674331	÷60
(19.)	7825687	÷64	(20.)	3795469	÷70
(21.)	3754329	÷80	(22.)	6598769	÷84
(23.)	8791605	÷88	(24.)	7654325	÷96
(25.)	3765897	÷23	(26.)	4613578	÷37
(27.)	5123495	÷41	(28.)	3954371	÷47
(29.)	3755123	÷234	(30.)	5764123	÷340
(31.)	34568135	÷357	(32.)	76549139	÷543
(33.)	29876533	÷6930	(34.)	56854327	÷7323
(35.)	95642371	÷8790	(36.)	34568795	÷9879

Answers to Examples in Division from 1 to 12.

(1.) 216258	(2.) 117263	(3.) 135939	(4.) 142783
(5.) 64289	(6.) 73832	(7.) 58166	(8.) 49428
(9.) $63003\frac{8}{9}$	(10.) $41622\frac{6}{11}$	(11.) $59737\frac{11}{12}$	(12.) $78047\frac{7}{12}$

Answers from 13 to 36.

(13.) 5)2366745÷15

3)473349

157783—*Answer.*

(14.) 3)7954326÷18

6)2651442

441907—*Answer.*

(15.) 6)6342576÷24

4)1057096

264274—Answer.

(16.) 6)6549372÷36

6)1091562

181927—Answer.

(17.) 9)4733491÷45

5)525943$\frac{4}{9}$

105188$\frac{31}{45}$—*Answer.*

(18.) 60)5674331÷60

94572$\frac{11}{60}$—*Answer.*

(19.) 8)7825687÷64

8)978210$\frac{7}{8}$

122276$\frac{23}{64}$—*Answer.*

(20.) 70)3795469÷70

54220$\frac{69}{70}$—*Answer.*

(21.) 80)3754329÷80

46929$\frac{9}{80}$—*Answer.*

(22.) 7)6598769÷84

12)942681$\frac{2}{7}$

78556$\frac{65}{84}$—*Answer.*

(23.) 8)8791605÷88

11)1098950$\frac{5}{8}$

99904$\frac{53}{88}$—*Answer.*

(24.) 8)7654325÷96

12)956790$\frac{5}{8}$

79732$\frac{53}{96}$—*Answer.*

(25.) 23)3765897(163734$\frac{15}{23}$ *Ans.*

23

146
138

85
69

168
161

79
69

107
92

15

(26.) 37)4613578(124691$\frac{11}{37}$ *Ans.*

37

91
74

173
148

255
222

337
333

48
37

11

(27.) 41)5123495(124963$\frac{12}{41}$ *Ans.*

41
102
82
203
164
394
369
259
246
135
123
12

(28.) 47)3954371(84135$\frac{26}{47}$ *Ans.*

376
194
188
63
47
167
141
261
235
26

(29.) 234)3755123(16047$\frac{125}{234}$ *Ans.*

234
1415
1404
1112
936
1763
1638
125

(30.) 34)576412,3(16953$\frac{103}{340}$ *Ans.*

34
236
204
324
306
181
170
112
102
103

(31.) 357)34568135(96829$\frac{188}{357}$ *Ans.*

3213

2438
2142

2961
2856

1053
714

3395
3213

182

(32.) 543)76549139(140974$\frac{257}{543}$ *Ans.*

543

2224
2172

5291
4887

4043
3801

2429
2172

257

(33.) 693,0)2987653,3(4311$\frac{1305}{6930}$ *Ans.*

2772

2156
2079

775
693

823
693

1303

(34.) 7323)56854327(7763$\frac{5878}{7325}$ *Ans.*

51261

55933
51261

46722
43938

27847
21969

5878

(35.) 879,0)9564237,1(10880$\frac{7171}{8790}$ *Ans.*

879

7742
7032

7103
7032

7171

Ans.

(36.) 9879)34568795(3499$\frac{2174}{9879}$

29637

49317

39516

98019

88911

91085

88911

2174

GREATEST COMMON MEASURE.

One number is said to be a measure or a factor of another, when it *divides it exactly* without a remainder.

Thus, 1, 2, 3, 4, 6, 12 are all measures or factors of 12. Unity, however, is not generally named among the divisors of a number. Any number which divides without remainder each of two or more numbers, is said to be a *Common Measure*, or common factor of those numbers; and of course the *greatest* number which so divides them is their *greatest* Common Measure, (G.C.M.)

Thus, 2 is the only common measure of 4 and 6; 3, 5, 15 are each of them common measures of 30 and 45, and 15 is their greatest common measure; 2, 7, 14 are each of them common measures of 14, 42, and 72, and 14 is their greatest common measure.

To find the greatest common measure of two numbers.

Rule.—Divide the *greater* by the *less*, and the *preceding divisor by the remainder*, and so on continually until there is *no remainder*, *the last divisor* will be the *greatest common measure* required.

EXAMPLE 1.

Find the G.C.M. of 3575 and 125455.

```
3575)125455(35
  10725
  -----
  18205
  17875
  -----
    330)3575(10
        330
        ---
        275)330(1
            275
            ---
             55)275(5
                275
                ---
                ...   55—Ans.
```

EXAMPLE 2.

Find the greatest common measure of the following—279 and 4185, and of 17 and 36.

```
279)4185(15
   279
   ---
  1395
  1395
  ----
  ....   279—Ans.
```

```
17)36(2
   34
   --
    2)17(8
      16
      --
       1)2(2
          2
          -
          .   1—Ans.
```

N.B.—The given numbers have no common measure but unity.

THE LEAST COMMON MULTIPLE.

One number is said to contain, or to be a *multiple* of another, when it can be divided by it without remainder.

Thus, 12 is a multiple of each of 1, 2, 3, 4, 6, 12, and any number is a *multiple of each of its* measures.

A *common multiple* of two or more numbers is one which *contains each of them;* and, of course, the *least* of such number is their least common multiple, (L.C.M.)

Thus, 6, 12, 18, &c., are all C.M. of 2 and 3; but 6 is their least common multiple. 12, 24, 36, 48, &c., are all common multiples of 2, 3, 4, 6, and 12; but 12 is their least common multiple.

Of course, a common multiple of *any* given numbers may be found by *multiplying* them all together. Thus, a common multiple of 6 and 8 is 48; of 4, 6, and 9 is 216. In practice, however, we require the *least common multiple*, especially in preparing Vulgar Fractions for Addition and Subtraction.

To find the least common multiple of two or more numbers.

Rule.—Set them in a line, and *strike out* any that are *contained* in any of *the others.* Divide those *not struck out* by *any* number that *will exactly* divide one of them; under any which it exactly measures place the *corresponding quotient;* under any which it partially measures (containing some factor common to it, but not *being itself wholly contained in it*), place the quotient obtained by dividing it by the common factor; and *under any which it does not measure* at all, repeat the number itself. Now treat the *new* line thus formed in the same manner as the *first*, and so on, until *all* the numbers left in any line have *no common measure* but unity. Then the continued product of the numbers in this line, and all the divisors, is the L.C.M. required of the given numbers.

Observe, it will *generally* be most convenient to take *large numbers*, if possible, for divisors, as *fewer lines* will thus be necessary, especially if such be chosen as contain themselves many simple factors. Thus, 12 contains the factors 2, 3, 4, 6, 12, and is therefore, *when possible*, a *very good* divisor to be employed.

EXAMPLE.

Find the least common multiple of

$$12)24 \cdot 16 \cdot \cancel{6} \cdot 20 \cdot \cancel{4} \cdot \cancel{8} \cdot \cancel{10} \cdot 30 \cdot \cancel{12} \cdot 25$$

$$\cancel{2} \quad 4 \quad 5 \quad \cancel{5} \quad 25$$

$$4 \times 25 \times 12 = 1200\text{—}\textit{Ans.}$$

VULGAR FRACTIONS.

A Fraction is a quantity which represents a part or parts of an integer or whole.

A *Vulgar* (that is, a common) fraction in its simplest form is expressed by means of *two* numbers placed one over the other with a dividing line *between* them.

The *lower* of these is called the *denominator*, and shews into how many *equal* parts the *whole* is *divided*. The *upper* is called the *numerator*, and shews *how many* of those parts are taken to *form* a fraction. Thus, $\frac{3}{4}$ denotes that the *whole* is divided into *four equal parts*, and that *three* of them are taken to *form* the fraction.

A *Proper* fraction is one whose numerator is *less* than the denominator, and which is itself, therefore, less than the whole in question, as $\frac{3}{8}$, $\frac{5}{7}$.

An *Improper* fraction is one whose numerator is *equal* to or greater than the denominator, and which is itself, therefore, equal to or greater than the whole in question, as $\frac{8}{8}$, $\frac{7}{7}$.

A *Mixed number* is one formed of a *whole number* and a *fraction*, as $2\frac{3}{8}$, $5\frac{3}{4}$.

A *Compound* fraction is a fraction of a fraction, as $\frac{3}{4}$ of $\frac{2}{3}$, $2\frac{1}{2}$ of $\frac{3}{7}$ of $3\frac{1}{4}$.

A *Complex* fraction is one in which either the numerator or denominator or both contain fractions, as $\frac{3\frac{1}{2}}{2}$, $\frac{2}{4\frac{2}{3}}$, $\frac{1\frac{1}{3}}{3\frac{3}{4}}$, $\frac{\frac{3}{4}\text{ of }3}{2\frac{1}{2}}$, &c.

Every *whole* number may be considered as a *fraction* whose denominator is *one;* thus, 6 is $\frac{6}{1}$. A fraction may be considered as expressing the division of the numerator by the denominator. Thus, $\frac{3}{4}$ expresses $3 \div 4$, for we should obtain the same whether we divide *one unit* into four equal parts, and then take *three* of these parts—that is, three-fourths of the one unit; or divide *three* units, each into four *equal parts*, and then take *one part* out of each four —*i.e.*, one-fourth of each unit, and therefore one-*fourth* of the whole *three units;* so that $\frac{3}{4}$ of 1, or $\frac{3}{4} = \frac{1}{4}$ of 3, or $3 \div 4$. For instance, $\frac{3}{4}$ of £1, which is 15 shillings, $= \frac{1}{4}$ of £3, which is also 15 shillings.

To reduce a whole number to a fraction with a given denominator.

Rule.—Multiply the number by the given denominator, and the result will be the numerator of the fraction required.

Example.—Reduce 5 to a fraction, with the denominator 6.

Since 1 contains 6 sixth parts, 5 contains 30 sixth parts, or $5 = \frac{30}{6}$.

Examples.

(1.) Reduce 8 and 27 to fractions, with denominators 5 and 27.

(2.) Reduce 34 and 135 to fractions, with denominators 11 and 17.

(3.) Reduce 6, 9, 12, and 20 to fractions, with denominator 15.
(4.) Reduce 25, 34, 70, and 111 to fractions, with denominator 34.

ANSWERS TO THE PRECEDING EXAMPLES.

(1.) $8 = \frac{8 \times 5}{5} = \frac{40}{5}$ *Ans.* $\quad 8 = \frac{8 \times 27}{27} = \frac{216}{27}$ *Ans.*

$27 = \frac{27 \times 5}{5} = \frac{135}{5}$ *Ans.* $\quad 27 = \frac{27 \times 27}{27} = \frac{729}{27}$ *Ans.*

(2.) $34 = \frac{34 \times 11}{11} = \frac{374}{11}$ *Ans.* $\quad 34 = \frac{34 \times 17}{17} = \frac{578}{17}$ *Ans.*

$135 = \frac{135 \times 11}{11} = \frac{1485}{11}$ *Ans.* $\quad 135 = \frac{135 \times 17}{17} = \frac{2295}{17}$ *Ans.*

(3.) $6 = \frac{6 \times 15}{15} = \frac{90}{15}$ *Ans.* $\quad 9 = \frac{9 \times 15}{15} = \frac{135}{15}$ *Ans.*

$12 = \frac{12 \times 15}{15} = \frac{180}{15}$ *Ans.* $\quad 20 = \frac{20 \times 15}{15} = \frac{300}{15}$ *Ans.*

(4.) $25 = \frac{25 \times 34}{34} = \frac{850}{34}$ *Ans.* $\quad 34 = \frac{34 \times 34}{34} = \frac{1156}{34}$ *Ans.*

$70 = \frac{70 \times 34}{34} = \frac{2380}{34}$ *Ans.* $\quad 111 = \frac{111 \times 34}{34} = \frac{3774}{34}$ *Ans.*

To reduce a mixed number to an improper fraction.

Rule.—Multiply the whole number by the denominator of the fractional part, add the result to the numerator of that part for the new numerator, and retain the same denominator.

EXAMPLES.

(1.) $7\frac{2}{3} = \frac{23}{3}$ for $7 = \frac{21}{3}$; and hence, $7\frac{2}{3} = \frac{21}{3} + \frac{2}{3} = \frac{23}{3}$

(2.) $1\frac{1}{8} = \frac{9}{8}$ (3.) $5\frac{4}{9} = \frac{49}{9}$ (4.) $3\frac{5}{7}$ (5.) $10\frac{2}{9}$

(6.) $221\frac{4}{11}$ (7.) $13\frac{15}{17}$ (8.) $32\frac{11}{13}$ (9.) $200\frac{27}{50}$

(10.) $71\frac{11}{12}$ (11.) $115\frac{13}{15}$ (12.) $128\frac{11}{18}$ (13.) $37\frac{15}{37}$

(14.) $200\frac{29}{30}$ (15.) $125\frac{24}{25}$ (16.) $514\frac{5}{16}$ (17.) $101\frac{10}{21}$

(18.) $719\frac{11}{12}$ (19.) $10\frac{113}{115}$ (20.) $17\frac{201}{239}$ (21.) $10\frac{213}{360}$

(22.) $111\frac{100}{111}$ (23.) $85\frac{89}{99}$

ANSWERS TO THE PRECEDING EXAMPLES, FROM 4 TO 23.

(4.) $3\frac{5}{7} = 3 + \frac{5}{7}$, and $3 = \frac{3 \times 7}{7} = \frac{21}{7}$, $\therefore 3 + \frac{5}{7} = \frac{21}{7} + \frac{5}{7} = \frac{26}{7}$ *Ans.*

(5.) $10\frac{2}{9} = 10 + \frac{2}{9}$, and $10 = \frac{10 \times 9}{9} = \frac{90}{9}$, $\therefore 10 + \frac{2}{9} = \frac{90}{9} + \frac{2}{9} = \frac{92}{9}$ *Ans.*

(6.) $221\frac{4}{11} = \frac{221 \times 11 + 4}{11} = \frac{2435}{11}$ *Ans.*

(7.) $13\frac{15}{17} = \frac{13 \times 17 + 15}{17} = \frac{236}{17}$ *Ans.*

(8.) $\frac{427}{13}$ *Ans.* (9.) $\frac{10027}{50}$ *Ans.* (10.) $\frac{863}{12}$ *Ans.*

(11.) $\frac{1738}{15}$ „ (12.) $\frac{2315}{18}$ „ (13.) $\frac{1384}{37}$ „

(14.) $\frac{6029}{30}$ „ (15.) $\frac{3149}{25}$ „ (16.) $\frac{8229}{16}$ „

(17.) $\frac{2131}{21}$ „ (18.) $\frac{8639}{12}$ „ (19.) $\frac{1263}{115}$ „

(20.) $\frac{4264}{239}$ „ (21.) $\frac{3813}{360}$ „ (22.) $\frac{12421}{111}$ „

(23.) $\frac{8504}{99}$ „

To reduce an improper fraction to a whole or mixed number.

Rule.—Divide the numerator by the denominator; the quotient will be a whole number, and the remainder, if any, the numerator of the fractional part of the mixed number required.

EXAMPLES.

$\frac{45}{9} = 5$ *Ans.* $\frac{113}{15} = 7\frac{8}{15}$ *Ans.*

Observe.—All improper fractions occurring in any question should (except the contrary be desired) be expressed as whole or mixed numbers.

Reduce the following to whole or mixed numbers:—

(1.) $\frac{37}{9}$ (2.) $\frac{79}{11}$ (3.) $\frac{313}{13}$ (4.) $\frac{2990}{23}$

(5.) $\frac{1023}{35}$ (6.) $\frac{3127}{43}$ (7.) $\frac{1210}{55}$ (8.) $\frac{2221}{87}$

(9.) $\frac{1247}{77}$ (10.) $\frac{3136}{95}$ (11.) $\frac{3000}{75}$ (12.) $\frac{3577}{102}$

(13.) $\frac{4148}{117}$ (14.) $\frac{4641}{221}$ (15.) $\frac{3135}{122}$ (16.) $\frac{6000}{375}$

(17.) $\frac{5434}{357}$ (18.) $\frac{6556}{401}$ (19.) $\frac{12321}{200}$ (20.) $\frac{23438}{333}$

ANSWERS TO THE PRECEDING EXAMPLES.

(1.) $\frac{3}{9}\frac{7}{} = 37 \div 9 = 4\frac{1}{9}$ *Ans.* (2.) $\frac{7}{11}\frac{9}{} = 79 \div 11 = 7\frac{2}{11}$ *Ans.*

(3.) $24\frac{1}{13}$ *Ans.* (4.) 130 *Ans.* (5.) $29\frac{8}{35}$ *Ans.*

(6.) $72\frac{31}{43}$ „ (7.) 22 „ (8.) $25\frac{46}{87}$ „

(9.) $16\frac{15}{77}$ „ (10.) $33\frac{7}{95}$ „ (11.) 40 „

(12.) $35\frac{7}{102}$ „ (13.) $35\frac{53}{117}$ „ (14.) 21 „

(15.) $25\frac{85}{182}$ „ (16.) 16 „ (17.) $15\frac{79}{357}$ „

(18.) $16\frac{140}{401}$ „ (19.) $61\frac{121}{200}$ „ (20.) $70\frac{128}{333}$ „

To multiply a fraction by any whole number or integer, either multiply the numerator or divide the denominator by it.

EXAMPLES.

(1.) $\frac{2}{15} \times 7 = \frac{14}{15}$, for in each of the fractions $\frac{2}{15}$, and $\frac{14}{15}$, the whole is divided into 15 equal parts, and 7 times as many of them taken in the latter case as in the former.

(2.) $\frac{7}{16} \times 4 = \frac{7}{4} = 1\frac{3}{4}$; for the whole being divided into 4 times as many equal parts in $\frac{7}{16}$ as it is in $\frac{7}{4}$, each of the parts in the latter is 4 times as great as in the former, and the same number of parts being taken in both cases, the latter fraction is therefore 4 times as great as the former.

(3.) $\frac{3}{5} \times 9 = \frac{27}{5} = 5\frac{2}{5}$ (4.) $\frac{7}{15} \times 4 = \frac{28}{15} = 1\frac{13}{15}$

(5.) $\frac{13}{27} \times 9 = \frac{13}{3} = 4\frac{1}{3}$ (6.) $\frac{15}{28} \times 7 = \frac{15}{4} = 3\frac{3}{4}$

Conversely.—To divide a fraction by any integer, either divide the numerator or multiply the denominator by it.

EXAMPLES.

(1.) $\frac{12}{13} \div 6 = \frac{2}{13}$ (2.) $\frac{15}{28} \div 5 = \frac{3}{28}$ (3.) $\frac{2}{3} \div 5 = \frac{2}{15}$

(4.) $\frac{7}{9} \div 6 = \frac{7}{54}$

EXAMPLES.

(1.) Multiply $\frac{35}{36}$ by 9, 12, 18, 25, and divide it by 5, 7, 8, 12.

(2.) „ $\frac{125}{144}$ „ 7, 8, 9, 16, and divide it by 5, 8, 12, 25.

(3.) „ $\frac{220}{693}$ „ the numbers 2, 3, 4, 5, 7.

(4.) *Divide* $\frac{220}{693}$ „ *the* numbers 7, 8, 9, 10, 11.

Answers to the preceding Examples.

(1.) $\frac{35}{36} \times 9 = \frac{35}{36 \div 9} = \frac{35}{4} = 8\frac{3}{4}$ *Ans.* (*Similarly*) $\frac{35}{36} \times 12 = \frac{35}{3} = 11\frac{2}{3}$ *Ans.*

and $\frac{35}{36} \times 18 = \frac{35}{2} = 17\frac{1}{2}$ *Ans.* $\frac{35}{36} \times 25 = \frac{35 \times 25}{36} = \frac{875}{36} = 24\frac{11}{36}$

Ans. $\frac{35}{36} \div 5 = \frac{35 \div 5}{36} = \frac{7}{36}$ *Ans.* (*Similarly*) $\frac{35}{36} \div 7 = \frac{5}{36}$ *Ans.*

$\frac{35}{36} \div 8 = \frac{35}{36 \times 8} = \frac{35}{288}$ *Ans.* So, $\frac{35}{36} \div 12 = \frac{35}{432}$ *Ans.*

(2.) $\frac{125}{144} \times 7 = \frac{875}{144} = 6\frac{11}{144}$ *Ans.* $\frac{125}{144} \times 8 = \frac{125}{18} = 6\frac{17}{18}$ *Ans.* $\frac{125}{144} \times 9 = \frac{125}{16} = 7\frac{13}{16}$ *Ans.* $\frac{125}{144} \times 16 = \frac{125}{9} = 13\frac{8}{9}$ *Ans.* $\frac{125}{144} \div 5 = \frac{25}{144}$ *Ans.* $\frac{125}{144} \div 8 = \frac{125}{1152}$ *Ans.* $\frac{125}{144} \div 12 = \frac{125}{1728}$ *Ans.* $\frac{125}{144} \div 25 = \frac{5}{144}$ *Ans.*

(3.) $\frac{320}{693} \times 2, 3, 4, 5, 7 = \frac{640}{693}, \frac{320}{231}, \frac{1280}{693}, \frac{1600}{693}, \frac{320}{99}$ *Ans.*

(4.) $\frac{320}{693} \div 7, 8, 9, 10, 11 = \frac{320}{4851}, \frac{40}{693}, \frac{320}{6237}, \frac{32}{693}, \frac{320}{7623}$ *Ans.*

If the numerator and denominator of a fraction be both multiplied, or both divided by the same number, its value will not be altered.

Examples.

(1.) $\frac{5}{7} = \frac{10}{14} = \frac{15}{21} = \frac{20}{28}$ &c. (2.) $\frac{36}{48} = \frac{18}{24} = \frac{12}{16} = \frac{9}{12}$ &c.

For if the numerator be multiplied by any number, the fraction is multiplied by it; and if the denominator be multiplied by any number, the fraction is divided by it; and if any number be both multiplied and divided by the same number, its value is not altered. Similarly, when the numerator or denominator of a fraction are both divided by the same number.

To reduce a fraction to lower terms.

Rule.—Divide both the numerator and denominator by any common factors they contain.

Examples.

$5)\frac{270}{375} = 3)\frac{54}{75} = \frac{18}{25}$ $9)\frac{315}{378} = 7)\frac{35}{42} = \frac{5}{6}$

It thus appears that the value of a fraction is not altered by this process.

When a fraction is reduced as much as possible by such division, it is said to be reduced in its lowest term.

Observe—All fractions occurring in any question should (except *where the contrary is desired*) be expressed in their lowest terms.

Reduce the Following to their Lowest Terms.

(1.) $\frac{324}{720}$ (2.) $\frac{720}{864}$ (3.) $\frac{324}{396}$ (4.) $\frac{1584}{5940}$ (5.) $\frac{1296}{1620}$ (6.) $\frac{1452}{2178}$ (7.) $\frac{495}{1210}$ (8.) $\frac{1296}{1728}$ (9.) $\frac{1872}{2016}$ (10.) $\frac{990}{1935}$ (11.) $\frac{3000}{3375}$ (12.) $\frac{2592}{3456}$ (13.) $\frac{1485}{2160}$ (14.) $\frac{864}{3072}$ (15.) $\frac{3300}{4235}$ (16.) $\frac{6930}{8118}$ (17.) $\frac{5544}{6552}$ (18.) $\frac{7040}{7392}$ (19.) $\frac{11385}{16335}$ (20.) $\frac{22176}{23328}$

Answers to the Preceding Examples.

(1.) $\frac{324}{720} = \frac{54}{120} = \frac{9}{20}$ *Ans.* (2.) $\frac{720}{864} = \frac{60}{72} = \frac{5}{6}$ *Ans.* (3.) $\frac{324}{396} = \frac{27}{33} = \frac{9}{11}$ *Ans.* (4.) $\frac{1584}{5940} = \frac{264}{990} = \frac{24}{90} = \frac{4}{15}$ *Ans.* (5.) $\frac{1296}{1620} = \frac{108}{135} = \frac{12}{15} = \frac{4}{5}$ *Ans.* (6.) $\frac{1452}{2178} = \frac{132}{198} = \frac{22}{33} = \frac{2}{3}$ *Ans.* (7.) $\frac{495}{1210} = \frac{45}{110} = \frac{9}{22} =$ *Ans.* (8.) $\frac{1296}{1728} = \frac{108}{144} = \frac{9}{12} = \frac{3}{4}$ *Ans.* (9.) $\frac{1872}{2016} = \frac{156}{168} = \frac{13}{14}$ *Ans.* (10.) $\frac{990}{1935} = \frac{198}{387} = \frac{22}{43}$ *Ans.* (11.) $\frac{3000}{3375}(\times\frac{8}{8}) = \frac{24000}{27000} = \frac{8}{9}$ *Ans.* (12.) $\frac{2592}{3456} = \frac{216}{288} = \frac{3}{4}$ *Ans.* (13.) $\frac{1485}{2160} = \frac{165}{240} = \frac{11}{16}$ *Ans.* (14.) $\frac{864}{3072} = \frac{72}{256} = \frac{9}{32}$ *Ans.* (15.) $\frac{3300}{4235} = \frac{300}{385} = \frac{60}{77}$ *Ans.* (16.) $\frac{6930}{8118} = \frac{630}{738} = \frac{70}{82} = \frac{35}{41}$ *Ans.* (17.) $\frac{5544}{6552} = \frac{462}{546} = \frac{77}{91} = \frac{11}{13}$ *Ans.* (18.) $\frac{7040}{7392} = \frac{880}{924} = \frac{80}{84} = \frac{20}{21}$ *Ans.* (19.) $\frac{11385}{16335} = \frac{2277}{3267} = \frac{23}{33}$ *Ans.* (20.) $\frac{22176}{23328} = \frac{1848}{1944} = \frac{154}{162} = \frac{77}{81}$ *Ans.*

A fraction may be reduced immediately to its lowest term by dividing both its numerator and denominator by their greatest common measure. This process is generally longer than the other, and is therefore, if possible, avoided in practice. It is, however, sometimes the only way of reducing a fraction, when we are unable to detect by inspection the common factors of the numerator and denominator. Thus, we should not see perhaps that the fraction $\frac{2509}{4181}$ may be reduced to $\frac{23}{37}$ by dividing both its terms by 113, their greatest common measure.

Examples.

$^{179})\frac{4117}{9487} = \frac{23}{53}$ *Ans.* $\qquad$ $^{564})\frac{13536}{17484} = \frac{24}{31}$ *Ans.*

Reduce to the Lowest Terms the Following :—

(1.) $\frac{321}{749}$ (2.) $\frac{510}{1122}$ (3.) $\frac{299}{529}$ (4.) $\frac{1407}{4422}$ (5.) $\frac{1905}{3175}$ (6.) $\frac{1715}{2695}$ (7.) $\frac{6509}{7889}$ (8.) $\frac{1589}{2270}$ (9.) $\frac{8251}{14718}$ (10.) $\frac{3575}{4719}$ (11.) $\frac{1261}{44232}$ (12.) $\frac{10769}{20408}$

Answers to the preceding Examples.

(1.) $\frac{321}{749} = \frac{107 \times 3}{107 \times 7} = \frac{3}{7}$ *Ans.* (2.) $\frac{510}{1122} = \frac{51 \times 10}{102 \times 11} = \frac{51 \times 10}{51 \times 22} = \frac{5}{11}$ *Ans.* (3) $\frac{299}{529} = \frac{13 \times 23}{23 \times 23} = \frac{13}{23}$ *Ans.* (4.) $\frac{1407}{4422} = \frac{201 \times 7}{402 \times 11} = \frac{201 \times 7}{201 \times 22}$ $= \frac{7}{22}$ *Ans.* (5.) $\frac{1905}{3175} = \frac{381 \times 5}{635 \times 5} = \frac{127 \times 3}{127 \times 5} = \frac{3}{5}$ *Ans.* (6.) $\frac{1715}{2695} =$ $\frac{343 \times 5}{539 \times 5} = \frac{49 \times 7}{77 \times 7} = \frac{7}{11}$ *Ans.* (7.) $\frac{6509}{7889} = \frac{6509}{1127 \times 7} = \frac{6509}{161 \times 49} =$ $\frac{6509}{23 \times 343} = \frac{23 \times 283}{23 \times 343} = \frac{283}{343}$ *Ans.* (8.) $\frac{1589}{2270} = \frac{227 \times 7}{227 \times 10} = \frac{7}{10}$ *Ans.* (9.) $\frac{8251}{14718} = \frac{8251}{1338 \times 11} = \frac{8251}{223 \times 66} = \frac{223 \times 37}{223 \times 66} = \frac{37}{66}$ *Ans.* (10.) $\frac{3575}{4719}$ $= \frac{715 \times 5}{1573 \times 3} = \frac{143 \times 25}{143 \times 33} = \frac{25}{33}$ *Ans.* (11.) $\frac{1261}{44232} = \frac{1261}{5529 \times 8} =$ $\frac{97 \times 13}{97 \times 19 \times 24} = \frac{13}{456}$ *Ans.* (12.) $\frac{10759}{20405} = \frac{1537}{2915} = \frac{1537}{583 \times 5} = \frac{1537}{53 \times 55} =$ $\frac{53 \times 29}{53 \times 55} = \frac{29}{55}$ *Ans.*

Examples in Addition, Subtraction, and Division of Vulgar Fractions.

ADDITION.

Rule.—If the fractions have a common denominator, add all the numerators together, and place their sum over the denominator.

Note.—If the fractions have not a common denominator they must be reduced to one, and compound and complex must also be reduced to simple fractions.

Examples.

(1.) Add $\frac{1}{4}$ and $\frac{3}{4}$ together: $\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$ *Ans.*

(2.) Add $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{6}{10}$ to $2\frac{1}{8}$ of $\frac{8}{4}$: $\frac{1}{2} \times \frac{3}{4} \times \frac{6}{10} = \frac{18}{80}$, and $2\frac{1}{8}$ of $\frac{8}{4} =$ $\frac{17}{8} \times \frac{3}{4} = \frac{51}{32}$; then $\frac{18}{80} + \frac{51}{32} = \frac{576}{2560} + \frac{4080}{2560} = 1\frac{2096}{2560} = 1\frac{131}{160}$ *Ans.*

(3.) Add $\frac{3}{5}$ and $\frac{5}{6}$ together. $1\frac{13}{30}$ *Ans.*

(4.) Add $\frac{5}{8}$ and $7\frac{1}{2}$ and $\frac{1}{3}$ of $\frac{3}{4}$ together. $8\frac{33}{8}$ Ans.

SUBTRACTION.

Rule.—Prepare the fractions when necessary the same as for other operations, then subtract the one numerator from the other, and set the remainder over the common denominator, and reduce to lowest terms.

EXAMPLES.

What is the difference between $\frac{5}{6}$ and $\frac{1}{6}$?

Thus $\frac{1}{6}$ from $\frac{5}{6} = \frac{4}{6} = \frac{2}{3}$ *Ans.*

Subtract $\frac{3}{9}$ from $\frac{6}{8}$.

$$\left.\begin{array}{ll} 6 \times 9 & 54 \\ 3 \times 8 & 24 \\ 8 \times 9 & 72 \end{array}\right\} = \frac{54}{72} - \frac{24}{72} = \frac{30}{72} = \frac{5}{12}$$ *Answer.*

Subtract $\frac{5}{12}$ from $\frac{7}{13}$. $\frac{19}{156}$ *Answer.*

Subtract $\frac{3}{5}$ of $\frac{1}{20}$ from $\frac{2}{7}$ of $5\frac{1}{6}$ of 1. $\frac{3037}{2100} = 1\frac{937}{2100}$ *Answer.*

MULTIPLICATION.

Rule.—Prepare the fractions when necessary as previously required; multiply *all* the numerators together for a *new* numerator, and all the denominators together for a new denominator.

EXAMPLES.

(1.) What is the product of $\frac{3}{4}$ and $\frac{3}{9}$?

$\frac{3}{4} \times \frac{3}{9} = \frac{9}{36} = \frac{1}{4}$ *Answer.*

(2.) What is the product of 6 and $\frac{2}{3}$ of 5?

$6 \times \frac{2}{3}$ of $5 = 6 \times \frac{10}{3} = \frac{60}{3} = 20$ *Answer.*

(3.) What is the product of $\frac{2}{3}$, $3\frac{1}{4}$, 5 and $\frac{3}{4}$ of $\frac{3}{5}$?

$$\frac{2}{3} \times \frac{13}{\frac{4}{2}} \times \frac{5}{1} \times \left(\frac{3}{4} \text{ of } \frac{3}{5}\right) \frac{\overset{3}{\cancel{6}}}{\underset{4}{\cancel{20}}} = \frac{13}{2} \times \frac{3}{4} = \frac{39}{8} = 4\frac{7}{8}$$ *Answer.*

(4.) What is the product of 5, $\frac{2}{3}$, $\frac{2}{7}$ of $\frac{3}{5}$ and $4\frac{1}{6}$ $2\frac{8}{21}$ *Answer.*

DIVISION.

Rule.—Prepare the fractions when necessary as previously required, then divide the numerator by the numerator, and the *denominator by the* denominator, if they will exactly divide; but

if not, *invert* the terms of the divisor, and multiply the dividend by it as in Multiplication.

EXAMPLE.

Divide $\frac{25}{9}$ by $\frac{5}{3}$: $\frac{25}{9} \div \frac{5}{3} = \frac{5}{5} = 1\frac{2}{3}$ *Answer.*

„ $\frac{5}{9}$ „ $\frac{2}{15}$: $\frac{5}{9} \div \frac{2}{15} = \frac{5}{5} \times \frac{15}{2} = \frac{75}{18} = 4\frac{3}{18}$ *Answer.*

„ $\frac{7}{16}$ „ $\frac{3}{4}$. $\frac{7}{12}$ *Answer.*

„ $\frac{3}{5}$ „ 2. $\frac{3}{10}$ *Answer.*

„ $\frac{2}{3}$ of $\frac{1}{3}$ by $\frac{5}{7}$ of $7\frac{3}{5}$. $\frac{7}{171}$ *Answer.*

DECIMAL ARITHMETIC.

INTRODUCTION.

An acknowledged authority has said on this, the simplest art of working practical calculations, that "our engineers and machinists, instead of *thinking* in eighths, sixteenths, and thirty-seconds of an inch, it is desirable that they should think and *speak* in tenths, hundredths, and thousandths."

The decimal system is simply the reduction of lengthy calculations to a few figures; therefore its decided advantages to all classes is obvious, and should be practised thoroughly, so as to make Decimals the fundamental principle of all mechanical calculations. It is the *simplest of languages;* for is it not easier to say or write three decimal three seven five (3·375), than the terse idiom and lengthy expressions of three and three-eighths ($3\frac{3}{8}$)? The advantages of the Decimal system are infinite, permitting us to find the fraction to the greatest nicety, and is proved by practice to be not only the superior but the simplest method of working.

For this reason I have introduced into the screw-cutting portion of my work the Decimal system of calculating various examples. It but needs a fair trial in order to prove its material advantages, its merits of correctness, and, if I may use the term, its beauties. Decimal calculation has been justly compared to a bouquet of flowers of "infinite variety," without a "thorn of complexity" to puzzle or perplex the operator; whereas "Vulgar Fractions" has no such merit to claim our attention. This simple method of *always* working in *tenths* renders the system, by the mere introduction of a dot (·), in all respects as clear as the usual rules of common arithmetic. It is sincerely hoped, therefore, that it may be continually *practised, so* as to make Decimals the universal rule *for all calculations.*

DECIMAL FRACTIONS.

A Decimal Fraction derives its name from the Latin word *decem*, signifying "ten," which denotes the *nature* of its numbers, representing the parts of an *integral* quantity divided in a *tenfold proportion.*

In *Decimal* Fractions the integer, or whole thing, as a gallon, a pound, a yard, &c., is supposed to be divided into *equal parts* called *tenths*, those tenths into ten equal parts called hundredths, and those hundredths into ten equal parts called thousandths, and so on *without end;* so that a denominator of a decimal being known *always* to consist of a *unit*, with as *many cyphers* as the *numerator has places*, is therefore *never* expressed, being *understood* to be 10, 100, 1000, 10000, &c., according as the numerator consists of one, two, three, four, or more figures. Thus, instead of $\frac{2}{10}$, $\frac{24}{100}$, or $\frac{211}{1000}$, the numerators *only* are written, with a *dot* or comma before them —thus, ·2, ·24, ·211, &c.

If a unit or whole quantity of any description, as a gallon, a pound, a foot, &c., be *divided* into *ten* equal parts, the *decimal* represents as many of those parts as the decimal figure expresses—thus, ·7 means seven of those parts, or seven tenths; but if the decimal consisted of *two* figures, unity would be understood to be divided into a *hundred* equal parts, of which the decimal represents as many as the figure expresses—thus, ·65 means sixty-five hundredths; and if the decimal consisted of *three* figures, unity would be supposed to be divided into a *thousand* equal parts, of which the *decimal* represents as many as the number expresses—thus, ·625 is six hundred and twenty-five of those parts; or, suppose the decimal ·0625, unity would be understood to be divided into 10000 equal parts, and the value of decimal figures may be made more plain by means of the following

TABLE.

Tenths, ...	·5
Hundredths, ...	·56
Thousandths, ...	·567
Ten thousandths, ...	·5078
Hundred thousandths, ...	·56789

Thus, ·5 is read five tenths, ·56 five tenths and six hundredths, or *fifty-six hundredths;* ·567 is read as five tenths six hundredths *and seven thousandths,* and so on, as in the table.

Cyphers to the *right hand* of decimals cause no *difference* in their *value*, for ·5, ·50, ·500 are decimals of the same value, being each equal to $\frac{1}{2}$—that is, $\cdot 5 = \frac{5}{10}$, $\cdot 50 = \frac{50}{100}$, $\cdot 500 = \frac{500}{1000}$. But if cyphers are placed on the *left hand* of decimals, they *diminish* their value in a *tenfold* proportion for each cypher—thus, ·3, ·03, ·003 are three tenths, three hundredths, and three thousandths, and answer to the vulgar fractions $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$ respectively.

A whole number and decimal are thus expressed — 85·75, 85·04, &c.

Reduction of Decimals.

By reduction we change vulgar fractions, which may represent the *less* parts of coin, weight, measure, &c., into decimals, and thus find the value of any fraction given, because decimals *increase* their value towards the *left hand*, and *decrease* their value towards the *right hand*, in the same tenfold proportion with integers or whole numbers. They may be annexed to whole numbers, and worked in all respects as whole numbers; hence, if simple arithmetic be *well understood*, there is little more to be learned than the placing of the separating point, the rule for which ought to be *well attended to*.

To reduce a Vulgar fraction to a Decimal of an equal value.

Rule.—*Add* a cypher or cyphers to the numerator, and *divide* by the denominator; the quotient will be the decimal required.

Example.

Reduce $\frac{14}{32}$ to a decimal.

32)14·0000(·4375—*Reduction.*
128
120
96
240
224
160
160

Thus you may take *any* number of cyphers at pleasure, but their number will be best ascertained when the work is finished; then *you must have as many* decimal figures as you have taken annexed.

cyphers in dividing, and if there are *not so many* in the quotient you must *prefix* cyphers to the *left hand*—thus, $\frac{1}{32} = \frac{1{\cdot}0000}{32}$ ·03125.

Sometimes the quotient figures will *repeat* continually, as $\frac{2}{3} = \frac{2{\cdot}0}{3} = \cdot\dot{6}$; then it is called a *repetend*, and the first figure which repeats may be dashed, to distinguish it from a *terminate decimal*. Sometimes two, three, or more figures will repeat, as $\frac{12}{33}$; thus, $\frac{12{\cdot}00}{33} = \frac{4{\cdot}00}{11} = \cdot\dot{3}\dot{6}$: such are called *compound repetends* or *circulates*. The *first* and *last* of the repeating figures may be dashed.

To reduce the *less* parts of coin, weights, measures, &c., to decimals.

Rule.—Divide the *least* name by such number as will *reduce* it to the *next greater;* to the decimal so obtained *prefix* the given number of the *same name*, then *divide* by such number as will *reduce* it to the *next greater*, *always annexing cyphers to the dividend* as occasion may require. Thus proceed till it be reduced to the decimal of the *required integer;* or reduce the given parts to a simple quantity by reducing them to the lowest name mentioned, *annex cyphers* thereto, and *divide* by such number as will *reduce* it to the *name required*, or, in other words, reduce the given parts to a *vulgar fraction*, and *that* fraction to a decimal.

EXAMPLE.

Reduce 17s. 10½d. to the decimal of a pound sterling.

Here $\frac{1{\cdot}0}{2}$d. = ·5d., and $\frac{10 + {\cdot}5}{12}$s. = $\frac{10{\cdot}500}{12}$s. = ·875 shillings; then,

$\frac{17 + {\cdot}875}{20}$£ = $\frac{17{\cdot}8750}{20}$£ = ·89375£ is the decimal required.

Or, by second method, $\frac{17\text{s. } 10\frac{1}{2}\text{d.}}{£} = \frac{429}{480}$ halfpence = ·89375.

To find the value of any given decimal.

Rule.—Multiply the decimal given by the *number* of parts of the *next inferior denomination*, cutting off the decimals from the *product;* then multiply these remaining decimals by the *next inferior* denomination—thus proceeding to the *least denomination of the integer.*

Examples.

Required the value of ·89375 of a pound sterling.

·89375	£
20	
17·87500	shillings
12	
10·500	pence
4	
2·0	farthings

17 shillings, 10 pence, ½ penny—*Ans.*

Reduce ·625 of a hundredweight to its proper terms.

Here ·625 × 4 qrs. = 2·500 qrs., and ·5 × 28 lbs. = 14·0 lbs.; whence 2 qrs. 14 lbs. is the answer.

Addition of Decimals.

Rule.—Arrange the numbers under each other according to their several values, find the sum, as in addition of whole numbers, and cut off for decimals as many figures to the right hand as there are decimals in any one of the given numbers.

Example.

What is the sum of 23·45, 7·849, 543·2, 8·6234, 253·004?

23·45
7·849
543·2
8·6234
253·004
836·1264—*Ans.*

If any of the decimals be repetends; in adding, *increase* the *sum* of the *first* column by as many *units* as would arise from the addition of a further column, and there will be a circulate in the *sum beginning and ending like* the others. Thus—

·7500
·6̇666
·8̇888
·8750
·4̇444

3·6̇249̇ = 3·625 nearly.

2·5000000
3·6̇666666
7·6̇969696
14·3̇72̇3723

28·23̇ 60087̇

The repetend of ·6̇, the circulate 6̇ 9̇ and 3̇ 72̇ are continued till their periods begin and end together. It may easily be observed that there would be 2 to carry to the first column if it were carried any further.

Notice.—It is not always necessary to attend to the rule for repetends and circulates, three or four decimal figures, according to the rule, being sufficiently near exactness for common calculation.

Subtraction of Decimals.

Place the numbers directly under each other according to their several values, subtract as in whole numbers, and cut off for decimals as in addition.

Example.

Subtract 35·87043 from 132·005.

132·005
35·87043

96·13457

If both be single repetends make them end together, and if there be occasion to borrow at the first figure *borrow* 9 *instead of* 10. Thus—

·8̇3̇
·6̇6̇

·1̇6̇

If both be circulates, or one a repetend and the other a circulate, continue till both their periods begin and end together; then, if there should be occasion to borrow at the following figure, if continued further, deduct *one* from the subtrahend, and if the numbers be in *different* denominations *reduce* them till they be alike.

Subtract $\frac{8\,3\,4}{9\,9\,9}$ from $1\frac{2}{3}$, thus—

$$1\frac{2}{3} = 1\cdot\dot{6}\,6\dot{6}$$
$$\frac{8\,3\,4}{9\,9\,9} = \cdot\dot{8}\,3\dot{4}$$
$$\frac{9\,9\,9}{} \qquad \cdot\dot{8}\,3\dot{1}$$

Multiplication of Decimals.

Rule.—Place the factors under each other, and multiply them together as in whole numbers; then point off as many figures from the product (counting from right to left), as the sum of the decimal places in both factors, observing, if there be not enough to *annex* as many cyphers to the *left* hand of the product as will supply the deficiency.

Example.

Multiply ·4375 by ·125.

$$\begin{array}{r} \cdot 4375 \\ \cdot 125 \\ \hline 21875 \\ 8750 \\ 4375 \\ \hline \cdot 0546875 \end{array}$$

—*Ans.*

Here the product of ·4375 by ·125 is ·0546875, and as there are *three* places of decimals in the multiplier and *four* in the multiplicand, a cypher must be *added* on the *left* hand of the product, to *reduce* it to its proper terms.

If the multiplicand be a *repetend* or *circulate*, consider the increase that would arise to the first product if the multiplicand were continued further. Thus do with each product, make them begin and end together, and *add* them by the rule for adding *circulates*.

Rule.—To *contract* the operation so as to retain *only* as many decimals in the product as may be thought necessary, place the unit-figure of the multiplier under that figure of the multiplicand whose place is the *last* to be retained in the product, and dispose of the rest so that *they may all* stand in *contrary* order to that in *which they are usually* placed.

Multiply those figures of the multiplicand immediately above and to the left of each multiplying digit in the multiplier, increasing each product by the number which would have arisen had the next figure to the right of the multiplicand been multiplied, and setting the products down so that the right hand figures may be in one vertical line, as in the example below :—

EXAMPLE.

Multiply 27·14986 by 92·41035, retaining four decimals in the product.

COMMON WAY.	CONTRACTED METHOD.
27·14986	27·14986
92·41035	53·014·29
13574930	24434874
8144958	542997
2714986	108599
10859944	2715
5429972	81
24434874	14
2508·9280650510	2508·9280

DIVISION OF DECIMALS.

Rule.—Prepare your decimals as directed for Multiplication, divide as in whole numbers, *cut* off as many figures for decimals in the quotient as the number in the dividend *exceeds* the number in the divisor—namely, make the number of decimal figures in the divisor and quotient together *equal* to the number in the dividend.

EXAMPLE.

Divide 173·5425 by 3·75.

```
3·75)173·54250(46·278
     1500
     ----
     2354
     2250
     ----
     1042
      750
     ----
     2925
     2625
     ----
     3000
     3000
```

Example.

Divide 1·4850 by 247·5.

Thus, $\frac{1{\cdot}4850}{247{\cdot}5} = {\cdot}006$, and if there are not as many decimal figures in the dividend as in the divisor, it is necessary to annex a sufficient number of cyphers, and if there is no remainder you must add cyphers to the right hand of the quotient, till you have taken as many in the dividend as will make the decimal figures therein equal to those in the divisor; thus, $\frac{14856}{2{\cdot}476} = 6000$.

Common Fractions Reduced to Decimals.

$\frac{1}{4} = {\cdot}25$	$\frac{3}{4} = {\cdot}75$	$\frac{1}{7} = {\cdot}1428$
$\frac{1}{3} = {\cdot}333$	$\frac{1}{5} = {\cdot}2$	$\frac{1}{8} = {\cdot}125$
$\frac{1}{2} = {\cdot}5$	$\frac{1}{6} = {\cdot}166$	$\frac{1}{9} = {\cdot}111$
$\frac{2}{3} = {\cdot}666$		

Table of Fractional Parts of an Inch Divided into Thirty-two Parts—Likewise Foot of Twelve Inches Reduced to Decimals.

Parts of an Inch.	Decimals.	Parts of an Inch.	Decimals.	Parts of a Foot.	Decimals.
$\frac{7}{8} + \frac{3}{32} =$	·96875	$\frac{3}{8} + \frac{3}{32} =$	·46875	11″ =	·9166′
$\frac{7}{8} + \frac{1}{16} =$	·9375	$\frac{3}{8} + \frac{1}{16} =$	·4375	10″ =	·8333′
$\frac{7}{8} + \frac{1}{32} =$	·90625	$\frac{3}{8} + \frac{1}{32} =$	·40625	9″ =	·75′
$\frac{7}{8} =$	·875	$\frac{3}{8} =$	·375	8″ =	·6666′
$\frac{3}{4} + \frac{3}{32} =$	·84375	$\frac{1}{4} + \frac{3}{32} =$	·34375	7″ =	·5833′
$\frac{3}{4} + \frac{1}{16} =$	·8125	$\frac{1}{4} + \frac{1}{16} =$	·3125	6″ =	·5′
$\frac{3}{4} + \frac{1}{32} =$	·78125	$\frac{1}{4} + \frac{1}{32} =$	·28125	5″ =	·4166′
$\frac{3}{4} =$	·75	$\frac{1}{4} =$	·25	4″ =	·3333′
$\frac{5}{8} + \frac{3}{32} =$	·71875	$\frac{1}{8} + \frac{3}{32} =$	·21875	3″ =	·25′
$\frac{5}{8} + \frac{1}{16} =$	·6875	$\frac{1}{8} + \frac{1}{16} =$	·1875	2″ =	·1666′
$\frac{5}{8} + \frac{1}{32} =$	·65625	$\frac{1}{8} + \frac{1}{32} =$	·15625	1″ =	·0833′
$\frac{5}{8} =$	·625	$\frac{1}{8} =$	·125	$\frac{7}{8}$″ =	·072916′
$\frac{1}{2} + \frac{3}{32} =$	·59375	$\frac{3}{32} =$	·09375	$\frac{3}{4}$″ =	·0625′
$\frac{1}{2} + \frac{1}{16} =$	·5625	$\frac{1}{16} =$	·0625	$\frac{5}{8}$″ =	·052083′
$\frac{1}{2} + \frac{1}{32} =$	·53125	$\frac{1}{32} =$	·03125	$\frac{1}{2}$″ =	·04166′
$\frac{1}{2} =$	·5			$\frac{3}{8}$″ =	·03125′
				$\frac{1}{4}$″ =	·02083′
				$\frac{1}{8}$″ =	·010416′

AVOIRDUPOIS WEIGHT.

16 drams	=	1 ounce.
16 ounces	=	1 pound.
28 pounds	=	1 quarter.
4 quarters	=	1 hundredweight.
20 hundredweights	=	1 ton.

MEASURE OF LENGTH.

12 inches make 1 foot.
3 feet „ 1 yard.
1760 yards „ 1 mile.

MEASURE OF AREA.

144 square inches make 1 square foot.
9 square feet „ 1 square yard.

MEASURE OF SOLIDITY.

1728 cubic inches make 1 cubic foot.
27 cubic feet „ 1 cubic yard.

MEASURE OF CAPACITY.

4 gills make 1 pint.
2 pints „ 1 quart.
4 quarts „ 1 gallon.

REDUCTION.

Reduction is expressing measures of a higher order in units of measure of a lower order, and units of measure of a lower order in measures of a higher.

To express a measure of a higher order in units of measure of a lower,

Rule.—Multiply the measure of the higher order by the number of times its unit of measure contains the unit of measure of the next lower order, adding the number of units of the lower order in the proposed measure, if there are any; repeat the process for each *order of measure until* the proposed order of measure is produced.

Examples.

(1.) Reduce 3 tons 15 cwt. 1 qr. 3 lbs. to ounces.

Tons	Cwt.	Qr.	Lbs.
3	15	1	3
20			
75 or 60 + 15 cwt.			
4			
301 or 300 + 1 qr.			
16			
4819 or 4816 + 3 lbs.			
16			
28914			
4819			
77104 ounces.			

(2.) Reduce 3 gallons 2 quarts 1 pint to gills.

Gals.	Qts.	Pt.
3	2	1
4		
14		
2		
29		
4		
116 gills.		

To express units of measure of a lower order in measures of a higher.

Rule.—Divide the units of measure of the lower order by the number of times that its unit of measure is contained in the unit of measure of the next higher order; the remainder, if any, will give the units of measure in the lower order; repeat the process for each order of measure until the proposed order of measure is *produced.*

Examples.

(3.) Reduce 100560 inches to miles.

12)100560 inches.

3)8380 feet.

1760)2793 1 foot (1 mile 1033 yards 1 foot.
1760

1033 yards.

(4.) Reduce 10011 ounces to hundredweights.

16)10011

16)625 11 oz.

4)39 1 lb.

9 cwt. 3 qrs. 1 lb. 11 oz.

(5.) Reduce 1 mile 3 yards 2 feet to inches.
(6.) Reduce 3 tons 3 cwt. 3 qrs. 0 lb. 3 oz. to ounces.
(7.) Reduce £176, 15s. 4½d. to farthings.
(8.) Reduce 10027 farthings to pounds.
(9.) Reduce 1767 gills to gallons.
(10.) Reduce 96036 cubic inches to cubic yards.
(11.) Reduce 1783 square inches to square yards.
(12.) Reduce 101137 inches to miles.

Answers.

(5.) 63492 inches.
(6.) 114243 ounces.
(7.) 169698 farthings.
(8.) £10, 8s. 10¾d.
(9.) 55 gallons 1 pint 3 gills.
(10.) 2 cubic yards 1 cubic foot 996 cubic inches.
(11.) 1 square yard 3 square feet 55 square inches.
(12.) 1 mile 1049 yards 1 foot 1 inch.

COMPOUND ADDITION.

Compound Addition consists in forming one sum of two or more measures of the same kind, but containing different orders of measure.

Arrange the measures so that those of the same order may be in one vertical line; add those of the lowest order together, and reduce the sum to the measure of the next order, setting down the remainder under the line added—carry on the same process throughout.

EXAMPLES.

	Tons	Cwt.	Qrs.	Lbs.	Oz.
(1.)	2	3	1	3	2
	7	1	0	1	11
	5	10	3	15	0
	1	8	1	11	11
	3	2	2	13	10
	19	6	0	17	2

	£	s.	d.
(2.)	10	11	$10\frac{1}{2}$
	46	13	$3\frac{1}{4}$
	17	2	$9\frac{1}{2}$
	11	16	$1\frac{3}{4}$
	17	0	$0\frac{1}{2}$
	103	4	$1\frac{1}{2}$

	Gals.	Qts.	Pts.	Gills.
(3.)	2	3	1	3
	11	1	0	2
	7	0	0	3
	2	1	1	1
	11	2	0	3
	35	1	1	0

From these examples, others may be prepared.

COMPOUND SUBTRACTION.

Compound Subtraction consists in finding the difference between two measures of the same kind, but containing different orders of measure.

Arrange the measures so that the one containing the lesser measure of the highest order shall be under the other, and the others in the same vertical line as in Addition. Proceed as in Simple *Subtraction, borrowing* and returning a unit of a higher *order instead of a digit* representing 10, 100, 1000, &c.

Examples.

(1.) Subtract £7 10s. 10½d. from £8 0s. 0d.

£	s.	d.
8	0	0
7	10	10½
0	9	1½

Here we have to subtract 2 farthings from 0 farthings, which is impossible, but by borrowing 1 penny or 4 farthings, it can be performed, leaving ½d. remainder; to return the penny and continue the subtraction necessitates borrowing 1 shilling, which, when the penny has been returned, leaves 11d., and subtracting 10d. leaves 1d. To return the shilling necessitates borrowing £1, which, when the shilling has been returned, leaves 19 shillings, and subtracting 10 shillings leaves 9 shillings. On returning the pound there will be £7 subtracted from £7, leaving £0, as shewn.

(2.)

£	s.	d.
8	6	6½
4	2	3¼
4	4	3¼

(3.)

Tons	Cwt.	Qrs.	Lbs.	Oz.
2	19	3	15	14
1	17	2	13	11
1	2	1	2	3

(4.)

Tons	Cwt.	Qrs.	Lbs.	Oz.
10	3	2	0	12
9	2	3	2	13
1	0	2	25	15

(5.)

£	s.	d.
15	10	3½
10	11	4¼
4	18	11¼

COMPOUND MULTIPLICATION.

Compound Multiplication consists in the multiplication of a measure including measures of different orders, but of the same kind, by an abstract number.

Arrange the measure in its orders of magnitude, then multiply the measure of the lowest order by the number, and reduce the product to the next order of measure, retaining the remainder as forming the measure of the lowest order of the required product, carrying on the units of the high order as in Simple Multiplication; *proceed thus with each order.* Any order not represented must be *represented by a zero* (0).

Examples.

(1.) Multiply £1 and 11½d. by 9.

£	s.	d.	
1	0	11½	Here shillings are represented by 0. Multiply 2 far-
		9	things by 9, gives 18 farthings as the product, which
			being reduced to pence gives 4½d.; the ½d. is retained and
9	8	7½	4d. are carried on; multiply 11d. by 9, gives 99, and add

4d., gives 103d., which reduced to shillings becomes 8s. 7d., retain 7d. and carry on 8s.; multiply 0s. by 9 gives 0, and add 8s. gives 8s. which retain; multiply £1 by 9 gives £9, and as there was nothing to carry on, £9 is retained, making the answer £9, 8s. 7½d.

(2.) Multiply £17, 6s. 10¼d. by 106.

Here it is not easy to multiply by 106, but 106 equals 10 × 10 + 6, and £17, 6s. 10¼d. will have to be multiplied by 10, that product by 10; £17, 6s. 10¼d. will have to be multiplied by 6, and this product added to that already obtained, thus:—

£	s.	d.		£	s.	d.
17	6	10¼ × 6				
		10				
173	8	6½	= 10 times	17	6	10¼
		10				
1734	5	5	= 100 times	17	6	10¼
104	1	1½	= 6 times	17	6	10¼
1838	6	6½	= 106 times	17	6	10¼

COMPOUND DIVISION.

Compound Division consists in obtaining the quotient resulting from the division by an abstract number of measure including measures of different orders, but of the same kind.

Arrange the measure in its orders of magnitude. Then divide the measure of the highest order by the given number, retaining the quotient, as forming the measure of the highest order of the required quotient; reduce the remainder to the next lowest order of measure, including in the reduction any measure of that order included in the given measure; proceed thus to the lowest order, and the remainder, if any, should be placed in the numerator of a fraction, having the divisor for a denominator, representing a fraction of the *measure of the lowest order.*

EXAMPLE.

(1.) Divide £1067, 13s. $10\frac{1}{2}$d. by 106.

£ s. D.
106)1067 13 $10\frac{1}{2}$(10
106

7
20

106)153(1
106

47
12

106)574(5
530

44
4

106)178(1
106

72 or £10 1s. $5\frac{1}{4}$ $\frac{72}{106}$d.

$\frac{72}{106}$ represents, in its lowest terms, $\frac{36}{53}$ of a farthing.

(2.) Divide £777, 17s. $7\frac{3}{4}$d. by 100.

This may be done in the same manner as above, or £777, 17s. $7\frac{3}{4}$d. may first be divided by 10, and the quotient by ten, as $10 \times 10 = 100$.

£ s. D.
10)777 17 $7\frac{3}{4}$

10)77 15 $9\frac{0}{4}$ $\frac{7}{10}$

7 15 $6\frac{3}{4}$ $\frac{67}{100}$

Here in the second line there is 1d. over after 9d., this with 3 farthings makes 7 farthings, which is not divisible by 10, it is therefore represented $\frac{0}{4}$, and the 7 remainder being placed in the numerator, and 10 the divisor in the denominator gives the fraction $\frac{7}{10}$ of a farthing—the final remainder $\frac{67}{100}$ is obtained in the same manner.

RULE OF THREE.

Ratio has been defined as the relation of two quantities in respect of magnitude. Proportion may be defined as the equality of two ratios. Thus, $\frac{1}{3}$ as a fraction represents as 1 is to 3, or as 1 : 3 as a *ratio; also* $\frac{4}{12}$ *as* a fraction represents as 4 is to 12, or as 4 : 12 as

a ratio; and $\frac{1}{3} = \frac{4}{12}$, which placed in the form of a proportion, is as 1 : 3 = or : : (so is) 4 : 12.

The Rule of Three is employed to find the fourth term of a proportional when three are given. Each ratio which forms a proportion must consist of abstract numbers, as above, or else of magnitudes of the same kind. Thus, we cannot compare days with feet, but days must be compared with days, hours, seconds, &c., and feet with feet, yards, miles, &c.

In the Rule of Three it is not only necessary that the ratios shall refer to magnitudes of the same kind, but they must be reduced to magnitudes of the same order of magnitude. In the third term is to be placed the magnitude of the same kind as that which is sought, and the first term of the proportion must be so arranged as to bear the same ratio to the second which the third will bear to the fourth.

Suppose that 16 pigs cost £64, what will be the price of 73? Now it will occur to a person unacquainted with the Rule of Three, that if £64 is divided by 16 it will give the price of 1 pig, and this multiplied by 73 will give the price of 73 pigs.

There are questions, however, and particularly in Double Rule of Three, where this mode of procedure will not be so readily perceived. For example—

If 17 men perform a piece of work in 18 days, in how many days will 72 men perform it?

Now the quantity required is time or days, this, therefore, has to be placed in the third term; and we have next to consider whether the fourth term will be greater or less than the third, and place in the first term the less or greater of the remaining magnitudes. Now it is evident that if a few men can do a piece of work in a certain time, more men will be able to do it in less time, hence the greater of the remaining quantities will have to be placed in the first term. The second and third terms have then to be multiplied together, and divided by the first. Hence :—

Men. Men. Days.
As 72 : 17 : : 18 : Answer.

But if two ratios are both reduced to the same extent, they will still bear the same proportion to one another; the above proportion may therefore, dividing both ratios by 18, take the following form—

As $72 \div 18 : 17 : : 18 \div 18$: Answer.
Or 4 : 17 : : 1 : $4\frac{1}{4}$ days.

$$4)17$$
$$4\frac{1}{4}$$

As a second example: What will be the price of 72 cwt. of coals, if 15 tons 11 cwt. cost £18 10s. 6d.

It is evident that money is to be placed in the third ter that 72 tons 3 cwt. will cost more than 15 tons 11 cwt therefore the less of these, 15 tons 11 cwt., has to be pl the first term—the magnitudes have also to be reduced same order. Hence—

Tons	Cwt.	Tons	Cwt.	£	s.	d.		£	s.	d.
As 15	11 :	72	3 : :	18	10	6 :	*Answer* =	80	19	$0\frac{4}{7}$
20		20		20						
311		1443		370						
				12						
				4446						
				1443						
				13338						
				17784						
				17784						
				4446						
				311)6415578(20628						
				622						
				1955			d. 12)20628			
				1866			20)1719 0			
				897				£80	19	$0\frac{4}{7}$
				622						
				2758						
				2488						
				270						
				4						
				311)1080($\frac{4}{7}$						
				933						
				147						
				311						

DOUBLE RULE OF THREE.

Double Rule of Three may be considered as an extension of ordinary Rule of Three, as by its employment questions may be solved which could also be solved by two or more applications of ordinary Rule of Three.

(1.) If 6 men working 10 hours a day can perform a piece of work in 17 days, how many days would 7 men have to work working 9 hours a day?

Here, instead of having three quantities given from which to obtain a fourth, we have five given to obtain a sixth. In working out such questions we have to put the quantity sought in the third term; we have then to reason, as in ordinary Rule of Three, about the two similar quantities of the remaining four, considering the other two as constant; we have then again to reason about these last two, considering the former two as constant. We have then to obtain the product of the second and third terms, and divide by the product of the first. In the present question, as will appear immediately, there are two first and two second terms.

As, 7 men : 6 men :: 17 days
2 2
9 hours : 10 hours —
3 34
— 10
21 ——
21)340(16$\frac{4}{21}$ days
21
——
130
126
——
4
—
21

1st, How many days? Hence days must be placed in the third term. Next, If 6 men can perform a piece of work in a certain number of days, in how many days will 7 men perform it? Evidently in fewer days; hence we put the greater quantity 7 in the first term, and 6 the less in the second.

2nd, If a number of men working 10 hours a day can perform

a piece of work in a certain number of days, in how many days would they perform it working 9 hours a day? Evidently they would require a larger number of days; hence we place the smaller quantity 9 in the first term, and the larger 10 in the second.

(2.) If the carriage of 120 tons of goods for 100 miles cost £1000, what will be the carriage at the same rate of 55 tons for 180 miles?

As, ~~120~~ tons : 55 tons :: ~~£1000~~ : £825—*Ans.*
~~2~~ ~~10~~
~~100~~ miles : ~~180~~ miles 5
3

hence $55 \times 3 \times 5 = 825$.

Examples to Rule of Three.

(1.) If 20 men can earn £15, 5s. in a week, how much should 73 men earn in the same time? £55, 13s. 3d.—*Ans.*

(2.) If 70 tons of coal can be carried a mile for £8, 17s. 11d., what rate is it per ton? 2s. 6½d.—*Ans.*

(3.) If I can travel a distance of 210 miles for £1, 5s., what would it cost to travel 360 miles at the same rate?
£2, 2s. 10½–½d.—*Ans.*

(4.) If 7 men can reap a field in 39 hours, how many hours will 9 men take? 30 hours 20 minutes—*Ans.*

Examples to Double Rule of Three.

(1.) If 20 men can earn £45, 15s. in 30 days, how much will 45 men earn in 27 days? £92, 12s. 10½d.—*Ans.*

(2.) If 12 men can dig a trench in 10 days, how many men will be required to dig one 6 times as large in 12 days?
60 men—*Ans.*

(3.) If a first-class railway ticket is 2¾ times a third, and I can travel 370 miles third-class for £1, 10s. 10d., what will it cost to travel 555 miles first-class. £6, 7s. 2½d.—*Ans.*

OF THE SQUARE ROOT.

When a number is multiplied by *itself*, as 6 × 6, or 9 × 9, &c., it produces the *square* or *second* power of *that* number; and the number itself is called the *root* of that square. A root consisting of a single figure is found by inspection of the following table:—

Root, - - -	1	2	3	4	5	6	7	8	9
Squares, - -	1	4	9	16	25	36	49	64	81
Cubes, - -	1	8	27	64	125	216	343	512	729

To *extract* or find the square root of *any* number which consists of more figures than one.

Rule.—Make a point or *dot* over every *second* figure, commencing at the *right hand* of the integer figures, and at the left hand of the decimal figures, by which the given square will be pointed into periods of *two* figures each, except the *first* or *left* hand period, which will sometimes have but *one*.

The *unit* figure must *always* be the *latter* figure in the period, for the decimal point must be *between* the periods, and not in the *midst* of a period.

Find the *greatest* root in the *first* period, which write in the quotient or root, and the square thereof under the *same* period, *subtract* therefrom, and to the remainder *annex* the next period for a *dividend*. *Double* the *quotient* for a divisor, see how often the *divisor* is contained in the *dividend*, with this consideration, that the answer *must* be the *units*-figure of the divisor. Write the answer in the quotient, also in the unit place of the divisor; then *multiply* the divisor so completed by the *last* quotient figure, write the product under the dividend, and subtract therefrom; then to the remainder *annex* the next period for a *new* dividend.

Thus proceed with *every* period; and if there be *still* a remainder, *annex* pairs of cyphers for *additional periods* till you have a sufficient number of decimals in the root. Vulgar Fractions, &c., may be reduced to decimals. The periods, which are whole numbers, give whole numbers; and *decimal periods* give decimals in the root.

EXAMPLE.

What is the square root of 76176?

```
76176(276
4
---
47)361
329
---
546)3276
```

3276 = *i.e.*, the proof 276 × 276 = 76176.

(2.) Required the square root of ·000854.

```
·000854(·029        The Proof.
4                          ·029
---                        ·029
49)454                     ---
441                        261
---                        58
13                         ---
                           000841
                         +   13
                           ---
                           ·000854—Proof.
```

OF THE CUBE ROOT.

When a square is multiplied *again* by its *root*, as 6 × 6 × 6, it produces the *cube* or *third* power of that root. Single cubes are found by the inspection of the preceding table (page 51).

To extract the cube root of any number that consists of more than one figure.

Rule.—Point the given cube into periods of three figures, so that the unit-figure be the *last* in its period, then from the *first* period *subtract* the *greatest* cube it contains, put the root as a quotient, and to the remainder bring down the *next* period for a dividend. Find a *divisor* by multiplying the *square* of the root by 300; see how often it is contained in the *dividend*, and the answer gives the next figure in the root. Multiply the divisor by the *last* figure in the root. Multiply *all* the figures in the root by 30 except the *last*, and that *product* by the *square* of the last. Then cube the *final* figure in the root, *add* these *three last* found numbers together, and *subtract this sum from* the dividend; to the remainder bring down *the next period for* a new dividend, and proceed as before.

EXAMPLE.—Required the cube root of 444194947.

444194947(763
343

101194
First Divisor 95976

5218947
5218947 Second Divisor.

$7^2 \times 300 = 14700$	$76^2 \times 300 = 1732800$
6	3
88200	5198400
$7 \times 30 \times (6^2 =)\ 36 = 7560$	$76 \times 30 \times (3^2 =)\ 9 = 20520$
$6 \times 6 \times 6 = 216$	$3 \times 3 \times 3 = 27$
95976	5218947

Besides the preceding, there is another and perhaps a better way of extracting the cube root, and which I shall attempt to render as intelligible as possible by an explanation of the following example :—

Required the cube root of 926859375.

First cypher.	Second cypher.
0	0
9	81
9	81
9	162
18	24300
9	1939
270	26239
7	1988
277	2822700
7	14575
284	2837275
7	
2910	
5	
2915	

```
926859375(975
729

197859
183673

14186375
14186375
```

After having divided the number into periods of three figures each, as before, place before it *two* cyphers at moderate distances from each other, and from the number itself, as represented in the example; then find, by reference to the table (page 51), the *nearest* cube contained in 926, the *first* period, and place its root, which is 9, in the quotient, as the first figure of the required root. Place it also *under* the first cypher, and by going through the process of addition we obtain 9. Multiply this by 9, and place the product under the *second* cypher; then, by again going through the form of *addition* we get 81, which being *multiplied* by 9 becomes 729. This is placed under the *first* period of figures (926) and *subtracted* from it, and to the remainder the *second* period (859) is annexed. Then, recommencing at the column of the *first* cypher, we again place a 9, and *add* up. To the sum we add *another* 9, and obtain 27. We now *multiply* the 18 by 9, and place the *product* (162) under the 81 of the *second* column, and by addition we obtain 243. We now turn to the *first* column, and *annex* one cypher to the 27, making it 270; and in a similar manner annex *two* cyphers to the 243 of the *second* column. All this is done *preparatory* to finding the *second* figure of the quotient, or of the required root.

To find that figure we *divide* the number 197859 of the *third* column by 24300 of the *second* column. The quotient would appear to be 8; this, however, is found on trial to be *too great*, we therefore take 7, which answers. We *add* this 7 to the *first* column, and *multiply* the sum (277) by 7, placing the product (1939) in the *second* column. Then, by adding up, we obtain 26239, the product, of which by 7 we place in the *third* column and then *subtract*, and to the remainder we annex the *last* period of figures (375).

Recommencing at the *first* column, we add 7 to the 277, and to the sum (284) we add 7 again. We now multiply the sum (284) by 7, and place the product in the *second* column, and add up. Then to the 291 in the first column we annex *one cypher*, and to the 28227 in the *second* column *two* cyphers.

To find the third figure of the root we divide the number

(14186375) in the *third* column by the figures (2822700) in the *second* column, and the quotient is found to be 5. This we *add* to the *first* column, and multiply the sum (2915) by 5, and place the product under the 2822700 of the *second* column; upon adding these we obtain 2837275, whose product by 5 is placed under the 14186375 of the *third* column, and being exactly equal to it, there is *no* remainder upon subtracting: consequently the work is finished, and 975 is the cube root.

Notice.—The above method is not only useful for extracting the cube root, but also for that of any other root, attention being paid to the following directions:—

Instead of *dividing* the number whose root is to be extracted into periods of *three* figures each, we divide it into periods of as *many* figures each as *correspond* to the order of the root: as *three* cyphers for the *fourth* root, and *four* for the *fifth* root. It must be borne in mind, too, that every *new* figure which we place in the quotient must be *added* to the *first* column as *many times* as correspond to the *order of the root;* and that the number of products *added* to the *second* column be *one less* than the above number, and that added to the *third two less*, and so on.

Lastly, before finding a *new* figure for the quotient we annex one cypher to the number in the *first* column, two to that in the *second*, *three* to *that* in the *third*, and so on, until we arrive at the *last column*, where, instead of *annexing* cyphers, we bring down the numbers that *make up* the *next* period.

If, after *extracting* the root, we have a *remainder*, we can continue the quotient to decimals by *annexing* to the *remainder* as many *cyphers* as there are *figures* in a period, and then proceeding as before; and if the number whose root is to be extracted consists of a *whole* number and a *decimal*, we *divide* it into periods by commencing at the *units place*, and proceeding towards the *left hand* to divide the whole number, and towards the *right hand* to divide the decimals; and if the decimal do not contain a *sufficient* number of figures to make up the *last* period, we supply the *deficiency* with *cyphers*. The following example will make the method clear:—

Required the fourth root of 285762·321.

First cypher.	Second cypher.	Third cypher.	
0	0	0	
2	4	8	
2	4	8	
2	8	24	
4	12	32000	
2	12	7947	
6	2400	39947	
2	249	8721	
80	2649	48668000	
3	258	318321	
83	2907	48986321	
3	267	319243	
86	317400	49305564000	
3	921	64070168	
89	318321	49369634168	
3	922		
920	319243		
1	923		
921	32016600	285762·3210(23·12	
1	18484	16	
922	32035084	125762	
1		119841	
923		59213210	
1		48986321	
9240		102268890000	
2		98739268336	
9242		·3529621664	

To find the root of a number consisting of integers and decimals

Rule.—Multiply the difference between the root of the integral part of the given number, and the root of the next higher integer number by the decimal part of the given number, and add the product to the root of the integral part of the number given: the sum will be the root of the number required. Correct in all cases of the square root to 3 places, and in the cube root to 7, if the roots of the integral parts are correct to 4 places of decimals.

EXAMPLES.

(1.) Required the square root of 60·2.

$$\sqrt{61} = 7{\cdot}8102$$
$$\sqrt{60} = 7{\cdot}7460$$

Difference $\quad 0642$

$\times \quad 2$

Product $\quad = 01284$

Then $\quad + 7{\cdot}7460$

$$\therefore \sqrt{60{\cdot}2} = 7{\cdot}75884$$

As required correct 3 places of decimals.

(2.) Required the cube root of 843·75.

$$\sqrt[3]{844} = 9{\cdot}4503$$
$$\sqrt[3]{843} = 9{\cdot}4466$$

Difference $\quad 0037$

$\quad 75$

Product $\quad = 002775$

Then $\quad + 9{\cdot}4466$

$$\therefore \sqrt[3]{843{\cdot}75} = 9{\cdot}449375 \text{ (as required).}$$

If the square root is required correct to more places of decimals: the following rule is to be employed.

Rule.—Correct to 7 places of decimals:—Multiply the root of the nearest integer number by twice the difference, divide the product by three times the integer portion of the number added to the given number, and the quotient added to the root of the given number nearly gives the required root. The following is an *example of this rule:*—

$$\sqrt{60} = 7{\cdot}7460$$
$$\times \quad {\cdot}4 = {\cdot}2 \times 2$$

$60 \times 3) + 60{\cdot}2 = 240{\cdot}2)3{\cdot}09840({\cdot}01289 + 7{\cdot}7460 = 0{\cdot}75889 =$ the root required.

2402

6964
4804

21600
19216

23840

If the number consists entirely of decimals, the root will be decimals also.

INSTRUMENTAL ARITHMETIC; OR, UTILITY OF THE SLIDE RULE.

The instrument commonly called a Slide Rule is constructed of two pieces of boxwood, a foot in length each, and connected together by a joint, which enables them to be folded up lengthwise upon an edge. On one side the entire rule is divided, upon its outer edges, into inches and eighths, for the purpose of taking dimensions; and upon its inner edges there are several plane scales, each divided into twelve parts, which are designed for the reduction of dimensions for the purposes of drawing. On the other side there is a metallic slide, and four lines marked A, B, C, and D; the two middle lines, B and C, are doubled in their dimensions—that is, they proceed from 1 to 10 twice; and the fourth line, D, is a single one, running from 1 to 10, and is called the line of roots. The divisions on the first, second, and third lines are marked alike, each beginning at 1, which may be 1, 10, 100, 1000, &c.; or ·1, ·01, ·001, &c.; but whatever it is assumed to be, the second or middle line of these divisions must be taken at 10 times as many as the first, and the third line must be taken at 10 times as many as the second.

Numeration is the first practice to be acquired upon this instrument, for when that is thoroughly understood all other operations will become quite easy. It is first to be observed that the values *of the divisions upon* the rule are *all arbitrary*, and the value set

upon them *must* be such as will meet the requirements of the question which it is proposed to determine. The principal divisions, indicated by figures at 1, 2, 3, 4 and so on to 10, are termed *primes*, and the next division *tenths;* and these again are, or may be, sub-divided into 100 and 1000 parts. If the 1 (next to the joint) represents 1-tenth, then will the middle one be a unit or a whole number, and the other figures toward the right hand are likewise whole numbers, from the middle 1 to 10 at the end; but if the first 1 represent a unit, then the middle 1 will be 10, and the 10 at the right hand 100; if the first 1 represents 10 the middle one will be 100 and that at the right hand 1000, always increasing in a tenfold proportion, according to the value set upon the first figure, or 1. The figures between 1 and 10 are designated after the same manner—that is, if 1 at the beginning is 1-tenth, 2 will be 2-tenths, and the next 2 towards the right hand 2 units (but if 1 at the beginning is 1 unit, then 2 will also be 2 units), and the other 2 will be 20 if the first one is called 10; then 2 will be 20, and the next 2 is 200, &c. When the 1 at the left hand is taken at ·1, or 1-tenth, the line is read in the following order:—1, 2, 3, 4, 5, 6, 7, 8, 9-tenths unity, or 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. When a higher value is set on them they will read thus (beginning next to the joint):—10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300, and so on to 1000.

To multiply two numbers, set 1 on A (first line) to either of the given numbers on B (second line), then at the other number on A (first line) will be found the product on B (second line).

EXAMPLE.

Multiply 12 by 25.

Under 1 on A put 12 on B, and under 25 on A is 300 on B.

Note.—If the third term runs beyond the end of the line, look for it on the first radius or other part of the line, and increase it 10 times.

To divide one number by another.

Set 1 on A to the divisor on B, then at the dividend on B will be found the quotient on A.

EXAMPLE.

Divide 300 by 25.

Under 1 on A put 25 on B, and at 300 on B is 12 on A.

Note.—If the dividend runs beyond the end of the line diminish it 10 or 100 times, as may be required to make it fall upon A, and *increase the quotient accordingly.*

To ascertain a fourth proportional.

Set the first term upon A to the second on B, then at the third on A is the fourth on B.

Example.

What is the fourth proportional to 8, 20, and 30?

Under 8 on A put 20 on B, then at 30 on A is 75 on B.

Rule of Three Direct.

In the rule of three direct there are three numbers given to find a fourth that shall have the same proportion to the third as the second has to the first. The operation is as the first term upon A is to the second on B, so is the third term upon A to the fourth upon B; or bring the first term upon B to the second on A, then against the third upon B is the result upon A.

Example.

If a man can walk 20 miles in 5 hours, how long will he require to walk 125 miles at the same rate?

Over 20 upon B put 5 upon A, and against 125 upon B is 31·25, the result upon A.

Involution, or obtaining the power, and evolution, or extracting the root of numbers, are very conveniently performed on the engineer's slide rule, for when the slide is set straight at both ends C is a line of squares and D is a line of roots; consequently, against *any* number upon D is its square upon C, and against any number upon C is its root upon D.

Examples.

(1.) What is the square of 16?

Opposite 16 upon D is 256, the square number upon C.

(2.) Required the square root of 625.

Opposite 625 upon C is 25 upon D, the root required.

The cube of a number is found by *inverting* the slide and setting the number to be cubed upon B to the same number upon D, and against 1 or 10 upon D is the cube number upon B. To obtain the cube root set the number on B to 1 or 10 upon D, and where *two* numbers of *equal* value meet upon the lines B and D is the *root required.*

EXAMPLE.

(1.) Required the cube of 9.

Set 9 upon B to 9 upon D, and against 10 upon D is 729 upon B.

(2.) Required the cube root of 343.

Set 343 upon B to 10 upon D, and against 7 upon B is 7 upon D, the root required.

These lines also serve to *multiply* the square of *any* number any number of times, thus:—

To find the product of 6 times 6 multiplied by 3.

Set 3 upon B to 6 upon D, and against 10 upon D is 108 upon B.

PART II.—GEOMETRY.

DEFINITIONS.

A *Point* has position but not magnitude.

A *Line* is length without breadth, and either right, curved, or mixed.

A *Right Line* is the shortest distance between two points.

A *Curved Line* is one that continually changes its direction.

A *Mixed Line* is composed of a right and a curved line.

A *Superficies* has length and breadth only, and is plane or curved.

A *Solid* has length, breadth, and thickness.

An *Angle* is the opening of two lines having different directions, and is either right, acute, or obtuse.

When one straight line falling on another straight line makes the adjacent angles equal to one another, each of these angles is called a *Right Angle;* and the straight line which stands on the other is called a *Perpendicular* to it.

An *Acute Angle* is less than a right angle.

An *Obtuse Angle* is greater than a right angle.

A *Triangle* is a figure of three sides.

An *Equilateral Triangle* has all its sides equal.

An *Isosceles Triangle* has two of its sides equal.

A *Scalene Triangle* has all its sides unequal.

A *Right-angled Triangle* has one right angle.

An *Obtuse-angled Triangle* has one obtuse angle.

An *Acute-angled Triangle* has all its angles acute.

A *Quadrilateral* is a figure of four sides, and has the following particular designations, namely:—

A *Parallelogram,* having its opposite sides parallel.

A *Square,* having length and breadth equal.

A *Rectangle,* a parallelogram having a right angle.

A *Rhombus,* or lozenge, having equal sides, but its angles not right angles.

A *Rhomboid,* a parallelogram, its angles not being right angles.

A *Trapezium,* having unequal sides.

A *Trapezoid,* having only one pair of opposite sides parallel.

Note.—A *Triangle* is sometimes called a *Trigon,* and a *Square* a *Tetragon.*

Polygons are plane figures having more than four sides, and are

either *Regular* or *Irregular*, according as their sides and angles are equal or unequal, and they are named from the number of their sides or angles, thus:—

A Pentagon has five equal sides.
A Hexagon has six equal sides.
A Heptagon has seven equal sides.
An Octagon has eight equal sides.
A Nonagon has nine equal sides.
A Decagon has ten equal sides.
An Undecagon has eleven equal sides.
A Dodecagon has twelve equal sides.

A *Circle* is a plane figure bounded by a curved line, called the *Circumference* or *Periphery*, and such that all right lines from the circumference to a point, called the *Centre*, within the circle are equal.

A *Diameter* is a right line passing through the centre of a circle or sphere, and terminated at each end by the periphery or surface.

An *Arc* is any part of the circumference of a circle.

A *Chord* is a right line joining the extremities of an arc.

A *Segment* of a circle is any part bounded by an arc and its chord.

The *Radius of a Circle* is a line drawn from the centre to the circumference.

A *Sector* is any part of a circle bounded by an arc and its two radii.

A *Semicircle* is half a circle.

A *Quadrant* is a quarter of a circle.

A *Zone* is a part of a circle included between two parallel chords.

A *Lune* is the space between the intersecting arcs of two eccentric circles.

A *Tangent* is a right line that touches a curve without cutting it.

A *Cotangent* is the tangent of the complement of the arc.

A *Secant* is the line running from the centre of the circle to the extremity of the tangent of the arc.

A *Cosecant* is the secant of the complement of an arc, or the line running from the centre of the circle to the extremity of the cotangent of the arc.

A *Sine of an Arc* is a line running from one extremity of an arc perpendicular to a diameter passing through the other extremity, and the sine of an angle is the sine of the arc that measures that angle.

A *Versed Sine* of an arc or angle is the part of the diameter intercepted between the sine and the arc.

A *Cosine of an Arc or Angle* is the part of the diameter intercepted between the sine and the centre.

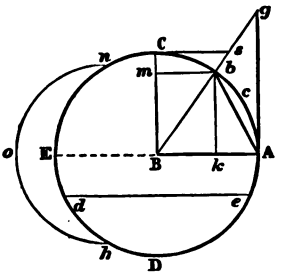
The *Coversed Sine of an Arc or Angle* is the part of the secondary radius intercepted between the cosine and the circumference.

The circumference of every circle is supposed to be divided into 360 equal parts, termed degrees; each degree into 60 minutes, and each minute into 60 seconds, and so on.

The *Complement of an Angle* is what remains after subtracting the angle from 90 degrees.

The *Supplement of an Angle* is what remains after subtracting the angle from 180 degrees.

To exemplify these definitions let ADEC, in the following diagram, be an assumed arc of a circle, described with the radius BA :—



A*cb*, an arc of the circle ACED.
A*b*, the chord of that arc.
BA, an initial radius.
BC, a secondary radius.
e*Dd*, a segment of the circle.
AB*b*, a sector.
ADE, a semicircle.
CBE, a quadrant.
A*ed*E, a zone.
noh, a lune.
B*g*, the secant of the arc A*cb*; written sec.
bk, the sine of the arc A*cb*; written sin.
A*k*, the versed sine of the arc A*cb*; written versin.
B*k* or *mb*, the cosine of the arc A*cb*.
A*g*, the tangent of the arc A*cb*.
*b*C, the complement, and *b*CE, the supplement of the arc A*cb*.
C*s*, the cotangent of the arc; written cot.
B*s*, the cosecant of the arc; written cosec.
*m*C, the coversed sine of the arc; written coversin.

The *Vertex* of a figure is its top or upper point. In conic sections it is the point through which the generating line of the conical surface always passes.

The *Altitude* or height of a figure is a perpendicular let fall from its vertex to the opposite side, called the base.

The *Measure of an Angle* is an arc of a circle contained between the two lines that form the angle, and is estimated by the number of degrees in the arc.

A *Segment of a Solid* is a part cut off by a plane parallel to the base.

A *Frustum* is the part remaining after the segment is cut off.

The *Perimeter of a Figure* is the sum of all its sides.

A *Problem* is something proposed to be done.

A *Postulate is* something required.

A *Theorem* is something proposed to be demonstrated.
A *Lemma* is something premised, to render what follows more easy.
A *Corollary* is a truth consequent upon a preceding demonstration.
A *Scholium* is a remark upon something going before it.

LENGTHS OF THE FOLLOWING ELEMENTS, RADIUS = 1.

Element.	Angle 45°.	Angle 60°.
Sine	·707107	·866025
Cosine	·707107	·5
Versed Sine	·292893	·5
Coversed Sine	·292893	·133975
Secant	1·414214	2·
Cosecant	1·414241	1·1547
Tangent	1·	1·73205
Cotangent	1·	·577349
Chord	·765366	1·
Arc	·785398	1·0472

SCALES.

To construct a Diagonal Scale upon any line, as AB.

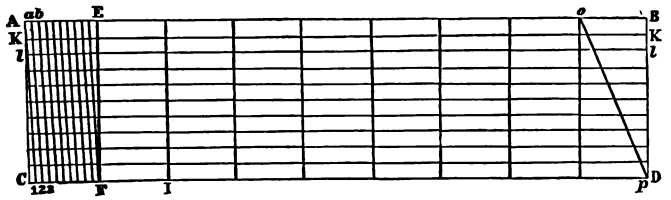


Fig. 1.

Draw eleven parallel equidistant lines.

Divide the line AB into as many divisions as there are hundreds of feet, spaces of ten feet, feet, or inches required.

Draw perpendiculars from every division to the lowest parallel line CD. Divide one of the divisions, AE, CF, into spaces of ten if for feet and hundredths, and twelve if for inches; draw the lines A1, *a*2, *b*3, &c., and they will complete the scale.

Thus, the line AB representing nine feet, A to E will measure one foot; A to *a*, C to 1, 1 to 2, &c., will measure 1-10th of a foot. *The several lines A1, a2,* &c., will measure upon the lines

KK, *ll*, &c., hundredths of a foot; and *op* will measure upon KK, *ll*, &c., divisions of tenths of a foot.

PRACTICAL GEOMETRY.

From any given point in a straight line to erect a perpendicular, or make a line at right angles with a given line.

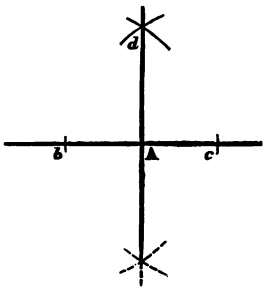


Fig. 2.

On each side of the point A, from which the line is to be made, take equal distances, as A*b*, A*c*, and from *b* and *c* as centres, with any distance greater than *b*A or *c*A, describe arcs cutting each other at *d*, then will the line A*d* be the perpendicular required.

When a perpendicular is to be made at or near the end of a given line.

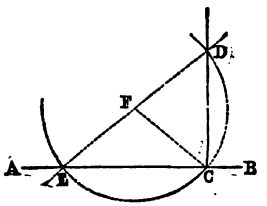


Fig. 3.

With any convenient radius and with any distance from the given line AB describe a portion of a circle, as ECD, cutting the given point in C; draw through the centre of the circle F the line EFD, and a line from the point C, cutting the intersections at D, is the perpendicular required.

To do the same otherwise.

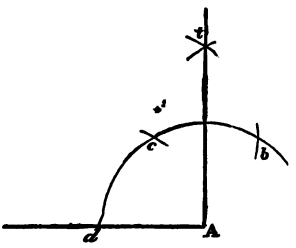


Fig. 4.

From the given point A, with any convenient radius, describe the arc *dcb;* from *d* with an equal radius cut the arc in *c*, and from *c* cut the arc in *b;* also from *c* and *b* as centres describe arcs cutting each other in *t*, then will the line A*t* be the perpendicular required.

Note.—When the three sides of a triangle are in the proportion of 3, 4, and 5 equal parts respectively, two of the sides (3 and 4) form a right angle.

To bisect any given angle.

From the point B as a centre, with any radius, describe an arc, as DE; and from D and E as centres, describe arcs cutting each other at F, then will the line BF bisect the angle as required.

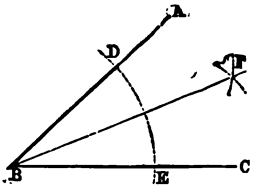


Fig. 5.

To find the centre of a circle that shall cut any three given points not in a direct line.

From the middle point *b* as a centre, with any radius, as *bc*, *bd*, describe a portion of a circle, as *csd*; and from *r* and *t*, the other points, as centres, with an equal radius, cut the portion of the circle in *cs* and *ds*; draw lines through where the arcs cut each other, and the intersection of the lines at *s* is the centre of the circle required.

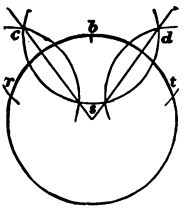


Fig. 6.

To find the centre of a given circle.

Bisect any chord in the circle, as AB, by perpendicular CD; bisect also the diameter ED in *f*, which is the centre of the circle required.

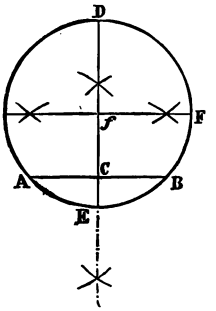


Fig. 7.

To find the length of any given arc of a circle.

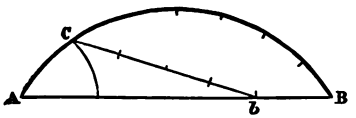


Fig. 8.

With a radius equal to $\frac{1}{4}$ the length of the chord of the arc AB, and from A as a centre, cut the arc in C; also from B as a centre, with equal radius, cut the chord in *b*; draw the line C*b*; and twice the length of the line is the length of the arc nearly.

Through any given point to draw a tangent to a circle.

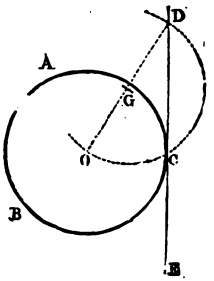


Fig. 9.

Let the given point be at D; draw the line DO to the centre, on which describe the semicircle DCO; draw the line DCE, cutting the circumference in C, which is the tangent required.

To inscribe a pentagon in a given circle.

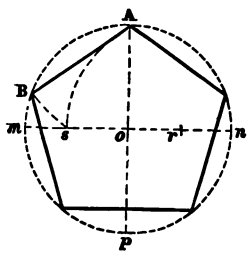


Fig. 10.

Draw the diameters A*p* and *mn* at right angles to each other, bisect *on* in *r*, and with *r*A describe A*s*; from A with A*s* describe *s*B. Join AB, and the distance is equal to one side of the pentagon required.

To circumscribe a pentagon about a given circle.

Let *s*, *r*, *v*, *m*, *n* be points of a pentagon inscribed in the circle. From the centre *o* draw *or*, *ov*, &c. Through *nm*, &c., draw AB, BC, &c., perpendicular to *on*, *om*, &c., and complete the figure.

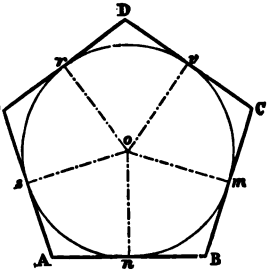


Fig. 11.

To inscribe any regular polygon in a given circle.

Divide any diameter, as AB, into as many equal parts as the polygon is required to have sides; from A and B as centres, with a radius equal to the diameter, describe arcs cutting each other in C; draw the line CD through the second point of division on the diameter, and the line DB is one side of the polygon required.

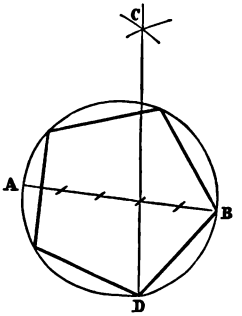


Fig. 12.

To describe an octagon on a given line.

From the points A, B, erect indefinite perpendiculars A*f*, B*e*; produce A, B, to *m* and *n*, and bisect the angles *m*A*o* and *n*B*p* with AH and BC. Make AH and BC equal to AB, and draw HG, CD, parallel to A*f*, and equal to AB.

From G and D as centres, with a radius equal to AB, describe arcs cutting A*f*, B*e*, in *f* and *e*. Join G*f*, *fe*, and *e*D, and the figure is complete.

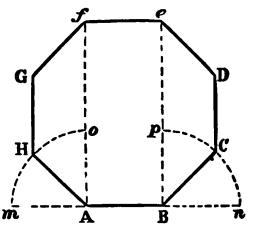


Fig. 13.

To describe a square about a given circle.

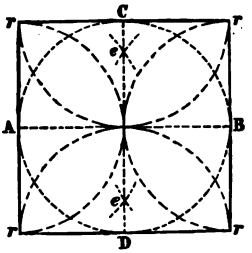


Fig. 14.

Draw the diameters AB, CD, at right angles. With A, B, C, D, as centres, and radius equal to that of the circle, describe circles intersecting at the points *r*; join these points, which will form the required square.

To describe a square in a given circle.

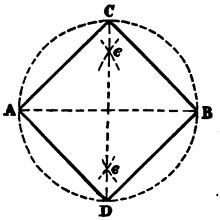


Fig. 15.

Draw the line AB through the centre of the circle; take any radius, as A*e*, and describe the arcs A*ee*, B*ee*; join *ee*, continuing the line to C and D; join AC, AD, &c., and the square is inscribed.

To describe an octagon in a given square.

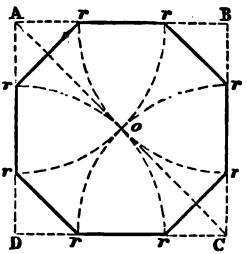


Fig. 16.

Let ABCD be the given square. Join A C, which bisect in *o*. Describe A*o*, B*o*, &c.; join the intersections *r*, *r*, *r*, *r*, &c., and the figure formed is the octagon required.

To inscribe an equilateral triangle in a given circle.

From any point A, with A*o* equal to the radius of the circle, describe *eoe*; from *e* and *e*, with a radius equal to that of the circle, describe *er*, *er*; join A*r*, *rr*, and *r*A, and the triangle is formed.

Note.—All figures of 10 or 20 sides are readily determined from the side of a pentagon, being halved or quartered; and in like manner, all figures of 6, 12, or 24 sides are readily determined from the radius of a circle, being equal to the side of a hexagon.

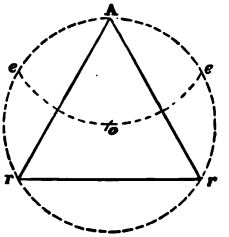


Fig. 17.

To describe an ellipse to any length and breadth required.

Let the longest diameter be CD, and the shortest EF. Take the distance C*o* or *o*D, and with it, from the points E, F, describe the arcs *h* and *f* upon the diameter CD. Insert pins at *h* and at *f*, and put a string round them of such a length that it will just reach to E or F. Introduce a pencil, and bearing upon the string, carry it around the centre *o*, and it will describe the ellipse required.

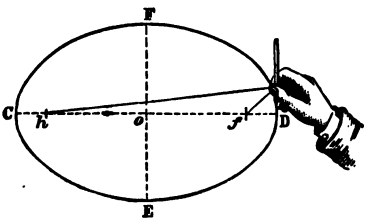


Fig. 18.

To describe an oval of any given length, without regard to breadth.

Let AB be the given length. Divide it into three equal parts, as ACDB. With the radius AC describe the circle AFDE, and from D BFCE. With EG and FK draw GH and JK, and the oval is described.

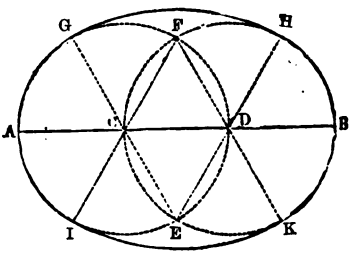


Fig. 19.

To ascertain the two diameters of an ellipse.

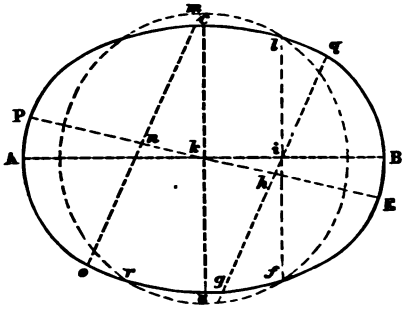


Fig. 20.

Let ABc*u* be the ellipse. Draw at pleasure two lines *gg*, *mo*, parallel to each other; bisect them in the points *hn*, and draw the line PE; bisect it in *k*, and upon *k* as a centre, describe a circle at pleasure, as *flr*, cutting the figure in the points *f*, *l*. Draw the right line *fl*; bisect it in *i*, and through the points *i*, *k* draw the greatest diameter AB, and through the centre *k* draw the least diameter *cu* parallel to the line *fl*.

To draw a spiral line about a given point.

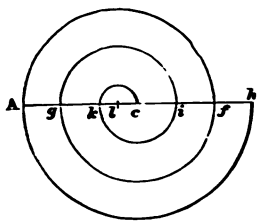


Fig. 21.

Let *c* be the centre; draw A*h* and divide it into twice the number of parts that there are to be revolutions of the line. Upon *c* describe semicircles *ki*, *gf*, A*h*; bisect *ck* in *l*, and upon *l* describe *kc*, *gi*, A*f*.

To describe a regular polygon of any required number of sides.

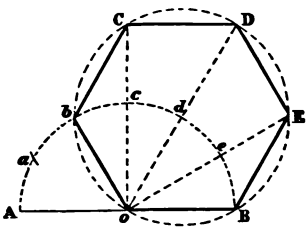


Fig. 22.

From the point *o*, with the distance *o*B, describe the semicircle B*b*A, which divide into as many equal parts, A*a*, *ab*, *bc*, &c., as the polygon is to have sides. Let a hexagon be required: From *o* to the second point of six divisions draw *ob*, and through the other points, *c*, *d*, and *e*, draw *o*C, *o*D, &c. Apply the distance *o*B from B to E, from E to D, from D to C, &c.; join these points, as *bC*, *CD*, *&c.*, and the figure is described.

To describe a Gothic arc.

Take the line AB. At the points AB draw the arcs B*a* and A*c*, and it will describe the arc required.

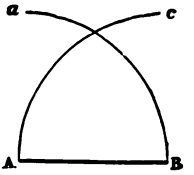


Fig. 23.

To describe an approximate semi-elliptic arc, the height being given.

Draw the diameter AB, and in the middle at K, erect the perpendicular K*o* equal to the height of the arc. Divide the perpendicular K*o* into two parts at *e*; continue the line AB on both sides at pleasure, and from the point K, with the distance K*o*, cut AB in *c* and *d*; through *ce*, *de*, draw *cef* and *deg* at pleasure; *d* and *c* are centres for the arcs A*g* and B*f*, and *e* the centre for the arc *gof*, which will form the arc required.

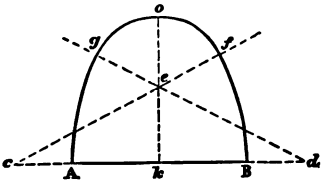


Fig. 24.

To describe an approximate elliptic arc, the chord and height being given.

Bisect AB at *c*; erect the perpendicular A*g*, and draw the line *g*D equal and parallel to A*c*. Bisect A*c* and A*g* in *r* and *n*; make *cl* equal to *c*D, and draw the line *lrg*; draw also the line *ns*D; bisect *s*D with a line at right angles, and cutting the line *c*D in *o*; draw the line *og*; make *cp* equal to *ck*, and draw the line *opi*. Then from *o* as a centre, with the radius *o*D, describe the arc *s*D*i*; and from *k* and *p* as centres, with the radius A*k*, *p*B, *describe the arcs As and Bi*, which will complete the arc required.

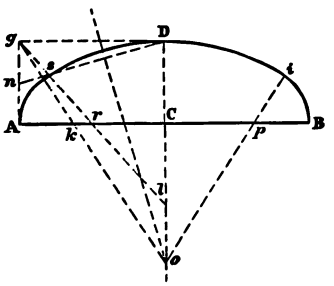


Fig. 25.

PART III.—MENSURATION.

Mensuration is the method of calculating the comparative magnitudes of figures, and is divided into two parts—viz., Mensuration of Superficies or Surfaces, and Mensuration of Solids. The magnitude of a surface is called its area, and is the space enclosed between its boundary lines. The magnitude of a solid body is called its solid contents, and is expressed in cubic feet, inches, &c.

To find the area of a parallelogram.

Rule.—Multiply the length by the perpendicular height, and the product will be the area.

To compute the area of a square, a rectangle, a rhombus or a rhomboid.

Rule.—Multiply the length by the breadth or height; or $l \times b$ = area, l representing the length and b the breadth.

Note.—The side of a square is equal to the square root of its area.

To find the area of a trapezoid.

Rule.—Add together the two parallel sides, multiply their sum by the height, and half the product is the area.

EXAMPLE.

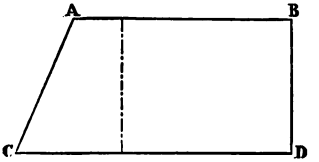


Fig. 26.

Required the area of a trapezoid whose sides AB and CD are 10·25 and 14·5, and height BD = 7·25. Here (AB + CD) × BD ÷ 2 = (10·25 + 14·5) × 7·25 ÷ 2 = 89·71875, the area of the trapezoid ABDC.

To compute the area of a triangle.

Rule.—Multiply the base by the height, and divide the product by 2; or, $\frac{b \times h}{2}$ = area, b representing the base and h the height.

Note.—The *hypothenuse* of a right angle is the side opposite to *the right angle.* The *perpendicular height* of a triangle is equal to *twice its area* divided by its base.

EXAMPLE.

The base *ab*, fig. 27, is 6 feet, and the height *cd* 4 feet; what is the area?

$4 \times 6 = 24$ and $24 \div 2 = 12$ square feet.

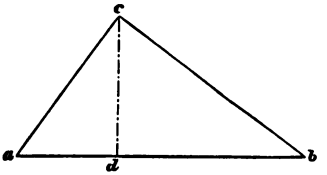


Fig. 27.

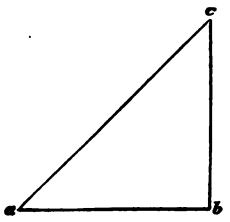


Fig. 28.

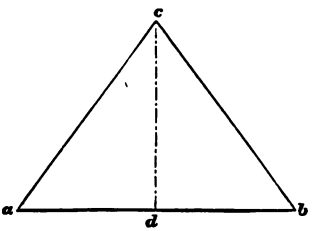


Fig. 29.

To find the area of any regular polygon.

Rule.—Multiply the sum of its sides by a perpendicular drawn from its centre to one of its sides, and take half the product for the area. Or multiply the square of the given side of a regular polygon by the numbers in the sixth column (*see following table,*) and headed "Area of the Polygon," opposite its name and given number of sides, and the product will be the area.

EXAMPLES.

(1.) Required the area of a regular pentagon, each side being 7″·5, and the perpendicular = 5·1614325 (see column 6 in the following table).

(2.) What is the area of a regular hexagon, each side being 8·75 in length? (See following table, column 6) Here $8{\cdot}75^2 \times 2{\cdot}598 = 199{\cdot}009375$, area—*Answer.*

The radius of a circle being given, to find the length of the side of a regular polygon inscribed in it.

Rule.—Multiply the given radius by that number in the fourth column of the following table which is opposite the given number *of sides, and the product* will be the length of the side.

TABLE OF MULTIPLIERS FOR REGULAR POLYGONS FROM 3 TO 50 SIDES.

No. of Sides	Name of Polygon.	Radius of the circumscribing circle, the inscribed being 1.	Side of the Polygon.	Radius of the circumscribing circle, the side being 1.	Area of the Polygon.
		A	B	C	D
3	Trigon	2·0000000	1·7320508	·5773503	·4330127
4	Tetragon	1·4142136	1·4142136	·7071068	1·0000000
5	Pentagon	1·2360680	1·1755706	·8506508	1·7204774
6	Hexagon	1·1547005	1·0000000	1·0000000	2·5980762
7	Heptagon	1·1099163	·8677676	1·1523825	3·6339126
8	Octagon	1·0823922	·7653668	1·3065630	4·8284272
9	Nonagon	1·0641778	·6840402	1·4619022	6·1818242
10	Decagon	1·0514622	·6180340	1·6180340	7·6942088
11	Undecagon	1·0422171	·5634652	1·7747331	9·3656415
12	Duodecagon	1·0352762	·5176380	1·9318517	11·1961524
13	Tridecagon	1·0299279	·4786312	2·0892913	13·1857718
14	Tetradecagon	1·0257169	·4450418	2·2469806	15·3345084
15	Pentadecagon	1·0223406	·4158234	2·4048672	17·6423629
16	Hexadecagon	1·0195912	·3901806	2·5629155	20·1093580
17	Heptadecagon	1·0173219	·3674990	2·7210970	22·7355038
18	Octadecagon	1·0154266	·3472964	2·8793853	25·5207681
19	Nonadecagon	1·0138273	·3291892	3·0377692	28·4652110
20	Eicosagon	1·0124651	·3128690	3·1962266	31·5687575
21	Uneicosagon	1·0112954	·2980846	3·3547557	34·8315014
22	Duoeicosagon	1·0102833	·2846296	3·5133383	38·2533531
23	Trieicosagon	1·0094016	·2723332	3·6719752	41·8344039
24	Tetreeicosagon	1·0086290	·2610524	3·8306488	45·5745246
25	Penteeicosagon	1·079480	·2506664	3·9893649	49·4738444
26	Hexeieicosagon	1·0073447	·2410734	4·1481206	53·5323900
27	Hepteicosagon	1·0068077	·2321858	4·3068951	57·7499409
28	Octeeicosagon	1·0063276	·2239290	4·4657083	62·1268039
29	Noneicosagon	1·0058966	·2162380	4·6245414	66·6627662
30	Triacontagon	1·0055083	·2090570	4·7833861	71·3577338
31	Untriacontagon	1·0051571	·2023366	4·9422682	76·2121205
32	Duotriacontagon	1·0048386	·1960344	5·1011600	81·2255440
33	Triatriacontagon	1·0045487	·1901122	5·2600645	86·3980681
34	Tetratriacontagon	1·0042841	·1845368	5·4189880	91·7298172
35	Pentriacontagon	1·0040420	·1792786	5·5779245	97·2207110
36	Hextriacontagon	1·0038198	·1743114	5·7368565	102·8704680
37	Heptriacontagon	1·0036155	·1696120	5·8958213	108·6797596
38	Octriacontagon	1·0034272	·1651586	6·0547953	114·6481841
39	Nonatriacontagon	1·0032533	·1609332	6·2137666	120·7755354
40	Tesseracontagon	1·0030922	·1569182	6·3727475	127·0620500
41	Untesseracontagon	1·0029429	·1530984	6·5317653	133·5082939
42	Duotesseracontagon	1·0028040	·1494602	6·6907572	140·1130227
43	Tritesseracontagon	1·0026749	·1459906	6·8497696	146·8771694
44	Tetratesseracontagon	1·0025544	·1426784	7·0087998	153·8007306
45	Pentesseracontagon	1·0024419	·1395130	7·1677935	160·8824925
46	Hextesseracontagon	1·0023367	·1364848	7·3268444	168·1245663
47	Heptesseracontagon	1·0022382	·1335852	7·4858794	175·5253176
48	*Octesseracontagon*	1·0021457	·1308062	7·6448940	183·0846240
49	*Nonatesseracontagon*	1·0020588	·1281404	7·8039765	190·8045888
50	*Pentecontagon*	1·0019772	·1255810	7·9629855	198·6818125

To compute the radius of a circle that contains a given polygon, when the length of a perpendicular from the centre alone is given.

Rule.—Multiply the distance from the centre to a side of the polygon by the unit in column A.

EXAMPLE.

What is the radius of a circle that contains a hexagon, the distance to the centre being 4·33 inches?

$$4{\cdot}33 \times 1{\cdot}1547 = 5 \text{ inches.}$$

To compute the length of a side of a polygon that is contained in a given circle, when the radius of the circle is given.

Rule.—Multiply the radius of the circle by the unit in column B.

EXAMPLE.

What is the length of the side of a pentagon contained in a circle 8·5 feet in diameter?

$$8{\cdot}5 \div 2 = 4{\cdot}25 \text{ radius, and } 4{\cdot}25 \times 1{\cdot}1756 = 5 \text{ feet.}$$

To compute the radius of a circumscribing circle, when the length of a side is given.

Rule.—Multiply the length of a side of the polygon by the unit in column C.

EXAMPLE.

What is the radius of a circle that will contain a hexagon, a side being 5 inches?

$$5 \times 1 = 5 \text{ inches.}$$

To compute the radius of a circle that can be inscribed in a given polygon, when the length of a side is given.

Rule.—Multiply the length of a side of the polygon by the unit in column E, following table.

EXAMPLE.

What is the radius of the circle that is bounded by a hexagon, its sides being 5 inches?

$$5 \times {\cdot}866 = 4{\cdot}33 \text{ inches.}$$

To compute the length of a side and radius of a regular polygon when the area alone is given.

Rule.—Multiply the square root of the area of the polygon by the multiplier in column F of the following table, for the length of the side; by the multiplier in column G for the radius of the circumscribing circle; and by the multiplier in column H for the *radius of the inscribed circle* or perpendicular.

No. of Side.	Name of Polygon.	Radius of inscribed circle.	Length of the side.	Radius of circumscribing circle.	Radius of inscribed circle.	Angle.	Angle of Polygon.	Tangents
		E	F	G	H			
3	Trigon.....	·2887	1·5197	·8774	·4387	120	60	·57735
4	Tetragon...	·5	1·	·7071	·5	90	90°	1·
5	Pentagon..	·6882	·7624	·6485	·5247	72°	108°	1·37638
6	Hexagon...	·866	·6204	·6204	·5373	60	120°	1·73205
7	Heptagon..	1·0383	·5246	·6045	·5446	51°25′	128°4′·7	2·07652
8	Octagon....	1·2071	·4551	·5946	·5493	45°	135°	2·41421
9	Nonagon...	1·3737	·4022	·588	·5525	40°	140°	2·74747
10	Decagon...	1·5388	·3605	·5833	·5548	36°	144°	3·07768
11	Undecagon	1·7028	·3268	·5799	·5564	32°43′	147°3′·11	3·40568
12	Dodecagon	1·866	·2989	·5774	·5577	30°	150°	3·73205

EXAMPLE.

The area of a square (tetragon) is 16 inches, what is the length of its side?

$$\sqrt{16} = 4, \text{ and } 4 \times 1 = 4 \text{ inches.}$$

Additional Uses of the Foregoing Table.—The seventh and eighth columns of the table facilitate the construction of these figures with the aid of a sector. Thus, if it is required to describe an octagon, opposite to it, in column seventh, is 45; then with the chord of 60 on the sector as radius, describe a circle, taking the length 45 on the same line of the sector; mark this distance off on the circumference, which being repeated around the circle will give the points of the sides.

The eighth column gives the angle which any two adjoining sides of the respective figures make with each other, and the ninth gives the tangent of the angle in column seven.

To compute the surface or linear edge of any regular solid body.

Rule.—Multiply the square of the linear edge, or the radius of the circumscribed or inscribed sphere, by the units in the following table, under the head of the dimension used :—

No. of Sides.	Names of Figures.	Surface.	Radius of circumscribed circle.	Radius of inscribed circle.	Linear edge by surface.
4	Tetrahedron,...	1·73205	1·63299	4·89898	·75984
6	Hexahedron,...	6·	1·1547	2·	·40825
8	Octahedron,....	3·4641	1·41421	2·44949	·53729
12	Dodecahedron,	20·64578	·71364	·89806	·22008
20	Icosahedron,...	8·66025	1·05146	1·32317	·33981

EXAMPLE.

What is the surface of a hexahedron or cube having sides of 5 inches?

$$5^2 \times 6 = 25 \times 6 = 150 \text{ square inches.}$$

To compute the linear edge when the surface alone is given.

Rule.—Multiply the square root of the surface by the multiplier under the head of linear edge by surface.

EXAMPLE.

What is the linear edge of a hexahedron, the surface being 6 square inches?

$$\sqrt{6} \times .40825 = 1.$$

The side of a regular polygon being given, to find the radius of the circumscribing circle.

Rule.—Multiply the given side by that number in the fifth column of the table which is opposite the number of sides, the product will be the radius of the circumscribing circle.

EXAMPLE.

Required the radius of the circle circumscribing an octagon, the length of its side being 7·6629502 inches. (See table, page 77.)

Here = 7·6629502 × 1·3065630 inches = 10·0121272 inches.

THE CIRCLE.

The circumference is assumed to be divided into 360 equal parts, termed degrees; each degree is divided into 60 equal parts, termed minutes; each minute into 60 equal parts, termed seconds; and each second into 60 equal parts, termed thirds.

To compute the circumference of a circle.

Rule.—Multiply the diameter by 3·1416; or, as 7 is to 22 so is the diameter to the circumference; or, as 113 is to 355 so is the diameter to the circumference.

EXAMPLE.

The diameter is $1\frac{1}{4}$ inches, what is the circumference?

$$1\frac{1}{4} \times 3.1416 = 3.927 \text{ inches.}$$

To compute the diameter of a circle.

Divide the circumference by 3·1416; or, as 22 is to 7 so is the *circumference to the diameter.*

Notice.—Divide the area by ·7854, and the square root of the quotient will give the diameter of the circle.

To compute the area of a circle.

Rule.—Multiply the square of the diameter by ·7854; or, multiply half the circumference by half the diameter; or multiply the square of the radius by 3·1416.

Notice.—The ratio of the diameter of a circle to its circumference has never yet been exactly determined; if the diameter be 1, then will the circumference be greater than 3·141592653, but less than 3·141592654. To four places of decimals we may take 3·1416, as being sufficiently exact for most practical purposes.

The Relative Proportions of the Circle: its Equal and Inscribed Squares.

1. The diameter of a circle	× ·8862	=	the side of a nearly = square.
2. The circumference of a circle	× ·2821		
3. The diameter of a circle	× ·7071	=	the side of an inscribed square.
4. The circumference of a circle	× ·2251		
5. The area of a circle	× ·6366	=	the area of an inscribed square.
6. The side of inscribed square	× 1·4142	=	the diameter of circumscribing circle.
7. The side of inscribed square	× 4·443	=	the circumference of circumscribing circle.
8. The side of a square	× 1·128	=	the diameter of a nearly = circle.
9. The side of a square	× 3·545	=	the circumference of a nearly = circle.

Examples Illustrative of the preceding Table.

(1.) The diameter of a circle is 12·5, required the side of a square nearly equal in area to the given circle.

Here, 12·5 × ·8862 = 11·07750.

(2.) The circumference of a circle being 53·4, required the side of a square nearly equal in area.

Here, 53·4 × ·2821 = 15·06414.

(3.) The diameter of a circle being 18, required the side of the greatest square that can be inscribed therein.

Here, 18 × ·7071 = 12·7278.

(4.) The circumference of a circle is 86, required the side of inscribed square.

Here, 86 × ·2251 = 19·3586.

(5.) The area of a circle being 371·5, required the area of the greatest square that can be inscribed within the circle.

Here, 371·5 × ·6366 = 236·49690.

(6.) The side of a square being 19·375, required the diameter of its circumscribing circle.

Here, 19·375 × 1·4142 = 27·4001250.

(7.) Required the circumference of a circle to circumscribe a square, each side being 19·375.

Here, 19·375 × 4·443 = 86·083125.

(8.) The side of a square being 13·5, required the diameter of a circle equal in area to the given square.

Here, 13·5 × 1·128 = 15·228.

(9.) The side of a square being 13·5, required the circumference of a circle equal in area to the given square.

Here, 13·5 × 3·545 = 47·8575.

Some of the Properties of a Circle.

1. It is the most capacious of all plane figures, or contains the greatest area within the same perimeter or outline.

2. The areas of circles are to each other as the squares of their diameters or of their radii.

3. Any circle whose diameter is double that of another contains four times the area of the other.

4. The area of a circle is equal to the area of a triangle whose base is equal to the circumference, and perpendicular equal to the radius.

5. The area of a circle is equal to the rectangle of its radius, and a right line equal to half its circumference.

6. The area of a circle is found by squaring the diameter and multiplying by ·7854, or by multiplying the circumference by the radius and dividing the product by two.

Examples.

(1.) Required the area of a circle, the diameter being 30″·5.

Here, $30''\cdot5^2$ (squared) × ·7854 = 730·61835, the area required.

(2.) What is the area of a circle when the diameter is one (1)?

In this case the circumference is 3·1416, half of which is 1·5708, and half of 1 = ·5; = then 1·5708 × ·5 = ·7854, the area.

Having the area of a circle given to find the diameter.

Rule.—Multiply the area by 452 and divide the product by 355, the square root of the product will be the diameter.

To find the area of a circular ring or space included between two concentric circles.

Rule.—*Add the inside* and outside diameters together, multiply

their sum by their difference, and by ·7854, and the product will be the area.

EXAMPLE.

The diameters of two concentric circles, AB and CD, are 10 and 6, required the area of the ring or space contained between them. Here, $(AB + CD) \times (AB - CD) \times \cdot7854 = (10 + 6) \times (10 - 6) \times \cdot7854 = 16 \times 4 \times \cdot7854 = 50\cdot2656$.

Definition.—The *cycloid* is a curve generated by the revolution of a point in the circumference of a circle on a plane.

Rule.—To ascertain the area of a cycloid, multiply the area of the generating circle by 3, and the product will give the area required.

EXAMPLES.

(1.) The generating circle of a cycloid has an area of 115·45 inches, what is the area of the cycloid?

$115\cdot45 \times 3 = 346\cdot35$ inches.—*Ans.*

(2.) The area of a circle describing a cycloid is 1·625 feet, what is the area of the cycloid in inches? 702 inches.—*Ans.*

(3.) The diameter of a circle describing a cycloid is 66·5 feet, what is the area of the cycloid in inches?

1500434·064 inches.—*Ans.*

Rule.—To ascertain the length of a cycloidal curve, multiply the diameter of the generating circle by 4, and the product will give the length of the curve.

EXAMPLES.

(1.) The diameter of the generating circle of a cycloid is 8 inches, what is the length of the curve? 32 inches.—*Ans*

(2.) The diameter of the generating circle is 20 inches, what is the length of the cycloidal curve? 80 inches.—*Ans.*

Note.—The curve of a cycloid is the line of swiftest descent; that is, a body will fall through the arc of this curve, from one point to another, in less time than through any other curved path.

MENSURATION OF SOLIDS.

By solids are meant all bodies, whether solid, fluid, or bounded space, that can be comprehended within length, breadth, and thickness.

To find the convex surface and cubical contents of a cylinder.

Rule.—Multiply the circumference of the base by the height of the cylinder, and the product is the convex surface. Or multiply

the area of the base by the height of the cylinder, and the product is the solid content.

EXAMPLES.

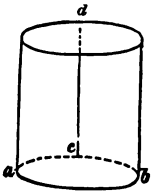


Fig. 30.

(1.) Required the convex surface of the cylinder *abcd*, fig. 30, whose base $ab = 32$ inches, and perpendicular height $bc = 6$ feet.

Here, $3{\cdot}1416 \times ab \times bc = 3{\cdot}1416 \times 32 \times 72$ inches $= 7238.2464$ square or superficial inches, and $7238.2464 \div 144 = 50{\cdot}2656$ superficial feet = Ans.

(2.) Required the solid content in cubic inches and cubic feet of the same cylinder as in Example 1.

Here, $(\overline{ab}^2 \times {\cdot}7854) \times bc = (\overline{32}^2 \times {\cdot}7854) \times 72 = 1024 \times {\cdot}7854 \times 72 = 57905{\cdot}9712$ cubic inches, and $57905{\cdot}9712 \div 1728 = 33{\cdot}5104$ cubic feet.

(3.) Suppose the cylinder *abcd* be intended to contain a fluid, and that the sides and bottom are each one inch in thickness, how many imperial gallons would it contain?

Here, $\{[(ab - 2 \text{ in.})^2 \times {\cdot}7854] \times (bc - 1 \text{ in.})\} = \{[(32-2)^2 \times {\cdot}7854] \times (72-1)\} = (30^2 \times {\cdot}7854 \times 71) = 900 \times {\cdot}7854 \times 71 = 50187{\cdot}06$ cubic inches $\div 277{\cdot}274 = 181$ imperial gallons.—*Answer.*

Note.—An imperial gallon contains 277·274 cubic inches.

To find the surface and solid content of a cone or pyramid.

Rules.—Multiply the circumference of the base by the slant height, and half the product will be the slant surface, to which add the area of the base, and the product will be the whole surface. Or multiply the area of the base by the perpendicular height, and one-third of the product will be the solid content.

EXAMPLES.

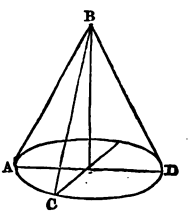


Fig. 31.

(1.) Required the convex surface of a cone whose base $AD = 20$ inches, and slant height $AB = 29{\cdot}5$.

Here, $\frac{1}{2}$ $(3{\cdot}1416 \times AD \times AB) = \frac{1}{2}$ $(3{\cdot}1416 \times 20 \times 29{\cdot}5) = 3{\cdot}141\,6 \times 10 \times 29{\cdot}5 = 926{\cdot}772$ square inches, which divided by $144 = 6{\cdot}436$ superficial feet.—*Answer.*

To find the surface of the frustum of a cone or pyramid.

Rule.—Multiply the sum of the perimeters of the two ends by the slant height, and half the product will be the slant surface, to which add the areas of the two ends, and the product will be the whole surface.

EXAMPLE.

Required the convex surface of the frustum of a cone ABCD, whose base AB = 20 inches, the slant height BD = 19, and top end CD = 11 inches.

Here, $\frac{1}{2}$ [(3·1416 × AB) + (3·1416 × DC)] × BC = $\frac{1}{2}$ [3·1416 × (AB + DC)] × BC = $\frac{1}{2}$ [3·1416 × (20 + 11)] × 19 = 1·5708 × 31 × 19 = 925·2012 square inches, which divided by 144 = 6·425 square feet. —*Answer.*

To find the solid content of the frustum of a cone.

Rule.—To the product of the diameter of the two ends, add the sum of their squares, multiply this sum by the perpendicular height, and by ·2618, the product is the solid content.

EXAMPLES.

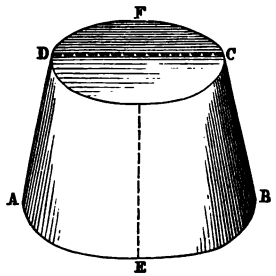


Fig. 32.

(1.) Required the solid content of the frustum of a cone (fig. 32), whose perpendicular EF taken = 18 inches, DC = 11 inches, and AB = 20 inches.

Here, (AB × DC + $\overline{AB^2}$ + $\overline{DC^2}$) × EF × ·2618 = (20 × 11 + $\overline{20^2}$ + $\overline{11^2}$) × 18 × ·2618 = (220 + 400 + 121) × 18 × ·2618 = 741 × 18 × ·2618 = 3491·8884 cubic inches, which divided by 1728 = 2·0208 cubic feet.—*Answer.*

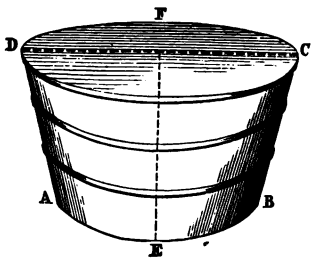


Fig. 33.

(2.) Required the content in imperial gallons of the inverted frustum of a cone ABCD, fig. 33, whose inner dimensions are EF = 3½ feet deep, AB = 18 inches diameter at bottom, and DC 22 inches diameter at the top.

Here, $\{(AB \times DC + \overline{AB}^2 + \overline{DC}^2) \times EF \times \cdot2618\} \div 277\cdot274 = \{(18 \times 22 + \overline{18}^2 + \overline{22}^2 \times 42 \times \cdot2618\} \div 277\cdot274 = \{(396 + 324 + 484) \times 42 \times \cdot2618\} \div 277\cdot274 = (1204 \times 42 \times \cdot2618) \div 277\cdot274 = 13238\cdot7024$ cubic inches $\div 277\cdot274 = 47\cdot746$ imperial gallons.—*Answer.*

To find the solid content of the frustum of a pyramid.

Rule.—To the sum of the areas of the two ends add the square root of their product, multiply this sum by the perpendicular height, and one-third of the product is the solid content.

EXAMPLE.

Required the solid content of the frustum of a pyramid (fig. 34) ABCD, whose perpendicular height = 24 inches, the area of the base AB = 144 square inches, and area of the top end DC = 64 square inches.—*Answer.*

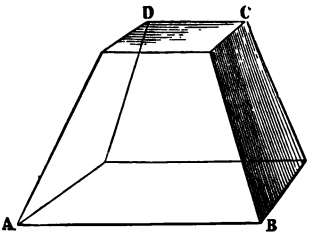


Fig. 34.

Here, $\{[\text{area at AB} + \text{area at DC} + \sqrt{(\text{area at AB} \times \text{area at DC})}] \times \text{perpendicular height}\} \div 3 = \{[144 + 64 + \sqrt{(144 \times 64)}] \times 24\} \div 3 = \{208 + \sqrt{(\overline{12}^2 \times \overline{8}^2)}\} \times 8 = \{208 + (12 \times 8)\} \times 8 = (208 + 96) \times 8 = 304 \times 8 = 2432$ cubic inches $\div 1728 = 1\cdot4074$ cubic feet.—*Answer.*

To find the convex surface and solid content of a sphere or globe.

Rule.—Multiply the square of the diameter by 3·1416, and the product is the convex surface; multiply the cube of the diameter by ·5236, and the product is the solid content.

EXAMPLES.

(1.) Required the convex surface of a sphere whose diameter = $25\frac{1}{2}$ inches.—*Answer.*

Here, $\overline{25\cdot5}^2 \times 3\cdot1416 = 2042\cdot8254$ square inches $\div 144 = 14\cdot1863$ square or superficial feet.

(2.) Required the solid content of a sphere whose diameter = $25\frac{1}{2}$ inches.—*Answer.*

Here, $\overline{25 \cdot 5}^3 \times \cdot 5236 = 8682 \cdot 00795$ cubic inches, which divided by $1728 = 5 \cdot 0243$ cubic feet.—*Answer.*

To find the convex surface and solid content of the segment of a sphere.

Rules.—Multiply the height of the segment by the whole circumference of the sphere, and the product is the curved surface. Add the square of the height to three times the square of the radius of the base; multiply that sum by the height, and by ·5236, and the product is the solid content.

EXAMPLES.

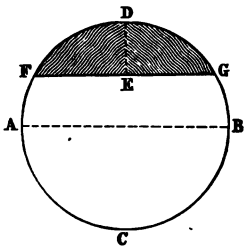


Fig. 35.

(1.) The diameter of the sphere ABCD = 20 inches, what is the convex surface of that segment of it whose height ED = 8 inches?

Here, $3 \cdot 1416 \times AB \times ED = 3 \cdot 1416 \times 20 \times 8 = 502 \cdot 656$ square inches, which $\div 144 = 3 \cdot 49$ superficial feet.—*Answer.*

(2.) The diameter of the base FG of the segment FGD = 18 inches, and perpendicular ED = 8, what is the solid content?

Here, $(\overline{ED}^2 + 3\,\overline{FE}^2) \times DE \times \cdot 5236 = (8^2 + 3 \times 9^2) \times 8 \times \cdot 5236 = (64 + 243) \times 8 \times \cdot 5236 = 307 \times 8 \times \cdot 5236 = 1285 \cdot 9616$ cubic inches, which divided by $1728 = \cdot 7442$ cubic feet.—*Answer.*

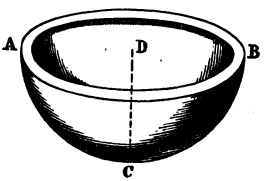


Fig. 36.

(3.) Suppose ABCD to be a sugar-pan, and that the diameter of the mouth AB is 4 feet, the depth DC being 25 inches, how many imperial gallons will it contain?

Here, $\{\overline{DC}^2 + (\text{half AB})^2 \times 3\} \times DC \times \cdot 5236 = \{25^2 + (24^2 \times 3)\} \times 25 \times \cdot 5236 = (625 + 576 \times 3) \times 25 \times \cdot 5236 = (625 + 1728) \times 25 \times \cdot 5236 = 2353 \times 25 \times \cdot 5236 = 30800 \cdot 77$ cubic inches, which divided *by* $277 \cdot 274 = 111 \cdot 084$ imperial gallons.—*Answer.*

To find the convex surface and solid content of a cylinder ring.

Rules.—To the thickness of the ring add the inner diameter, multiply the sum by the thickness and by 9·8696, and the product will be the convex surface. Or, to the thickness of the ring add the inner diameter, multiply that sum by the square of the thickness and by 2·4674, and the product will be the solid content.

EXAMPLES.

(1.) The thickness of a cylindrical ring = 2 inches, and inner diameter = 18 inches, required the convex superficies.

Here, $= (2 + 18) \times 2 \times 9{\cdot}8696 = 20 \times 2 \times 9{\cdot}8696 = 394{\cdot}784$ square inches, which $\div$ 144 = 2·742 square feet.

(2.) Required the solid content of the same ring.

Here, $(18 + 2) \times 2^2 \times 2{\cdot}4674 = 20 \times 4 \times 2{\cdot}4674 = 197{\cdot}392$ cubic inches, which $\div$ 1728 = ·114 cubic feet.

Notice.—A cubic foot is equal to 1728 cubic inches, or 2200 cylindrical inches, or 3300 spherical inches, or 6600 conical inches.

Also, the cubic foot being considered unity or 1, a cylinder 1 foot diameter and 1 foot in length = ·7854; a sphere 1 foot in diameter = ·5236; and a cone 1 foot in diameter at the base and 1 foot in height = ·2618.

CUTTING OF BOILER PLATES, MATERIALS FOR ROOFING, AND THE COVERING OF SOLIDS.

N.B.—Illustrating only such cases as are in general practice.

To ascertain the covering for a cone.

From B as a centre, with the radius BC, describe the arc of a circle C*d*; make C*d* equal in length to the circumference of the base of the cone (which can be found by a reference to the table of the "Circumferences of a Circle"), and join B*d*; then the figure determined by B*d*C will be that of the required surface of the cone.

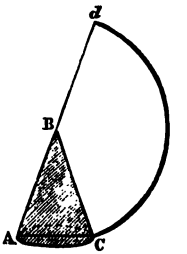


Fig. 37.

To ascertain the surface of an envelope for a frustum of a cone.

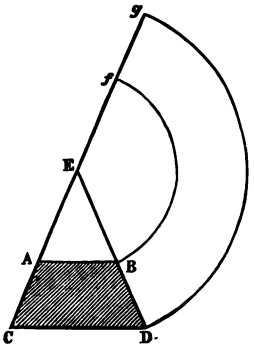


Fig. 38.

Produce AC, BD, until they meet at E; then from E as a centre, with the radii EA and EC, describe the arcs B*f* and D*g*, and set off D*g* equal to the circumference of the base of the cone, and join E*g*; then the figure BD*gf* will be that of the required surface of the frustum.

Method of finding the proper form of flat plate for an oblique section of a tube.

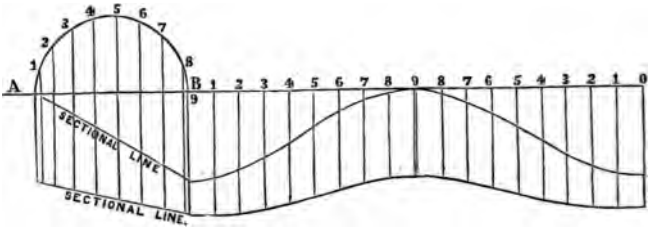


Fig. 39.

Take the diameter of tube AB and describe half the circumference; divide it into any number of equal parts, and set them out on a line continued on from the diameter as numbered 1, 2, 3, 4, &c.; draw the lines from the division of the circle at right angles across the diameter to intersect the line of section, and prick off the height of each line, as shewn. Draw a curved line through the pricked marks, which will give the section.

The covering for spheres, or any other curved figure, may be obtained in the same way.

To ascertain the surface of an envelope for a given cylinder.

Let ABCD be the given cylinder, having one of its ends at an *angle to its sides, as* AB. Upon the base CD draw a semicircle,

which divide into a number of equal parts (the more there are the more correct will be the result required), as 1, 2, 3, 4, 5, and 6; from the points draw the lines 1 1, 2 2, 3 3, &c., parallel to the plane of the cylinder; then set off, as in fig. 41, the line EFG, equal in length to the circumference of the base of the cylinder, or twice the semicircle CoD; upon EFG mark off on each side of F the

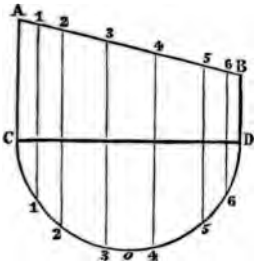


Fig. 40.

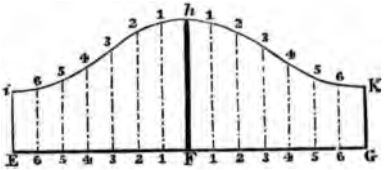


Fig. 41.

same number of equal distances as in the semicircle CoD, and draw lines, as F*h*, 1 1, 2 2, 3 3, &c., perpendicular to EFG; make F*h* equal in length to AC, and E*i*, G*k*, each equal to DB, and also each of the perpendiculars equal to the lines intercepted between AB and CD bearing the same figures, as 1 1, 2 2, 3 3, &c.; then a line traced through these points will form a side of the envelope, the whole of which is contained in the figure E*i*KG.

Note.—The above problem is a very useful one to boilermakers and coppersmiths, who are in the habit of making pipes to join each other at any given angle—the shape of the plate of metal for making which will be readily found by the rule preceding it.

COVERING OF CIRCULAR ROOFS, &c.

Circular roofs may be covered upon two different principles:—

First Method.—Assume the vertical section or axis to be divided into a number of small equal parts, and the roof or figure cut by planes through the points of division parallel to the base, and then consider the portions of the figure as so many frustums of cones; the surface of each frustum can then be determined by Rule 2, page 84.

Second Method.—Divide the circumference of the base into a

number of small equal parts, and assume sections to be made perpendicularly through these points of division; then estimate the surface of each of these divisions on the surface of the figure.

To cover a dome by the first method.

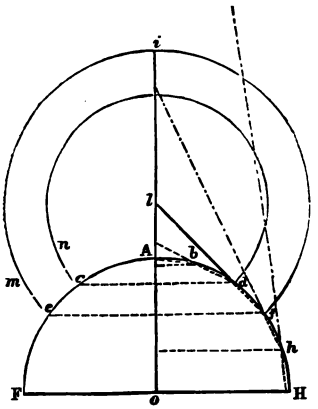


Fig. 42.

Let FAH, fig. 42, be the section of a dome; draw the axis *o*A produced to *i*; divide the curve of one-half the figure into equal parts, as *b*, *d*, *f*, and *h*; the width of these divisions being the width required by that of the material with which the dome is to be covered.

Produce H*h*, *hf*, *fd*, *db*, and *b*A severally, until they intersect the axis *o*A*i*; then (for example), from the point *l*, with the radii *lf* and *ld*, describe the curves *fm*, *dn*; then set off, as by Rule 2, the length of *fm* equal to the circumference of the base *fe*, and *dfmn* will be the form of the course to cover that of *dcef*.

In the same manner, the form of the coverings for the other portions can be found.

It will be observed that the finding of the centres for the curves of the lower courses of a dome of large diameter will be inconvenient, from the distances being frequently inaccessible with ordinary means.

The method, therefore, of finding these distances, when they are inconveniently accessible, is as follows:—

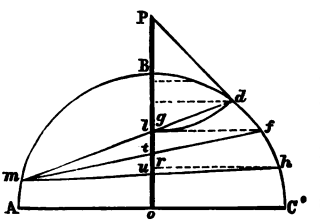


Fig. 43.

Let ABC, fig. 43, be the vertical section of a dome, and suppose *df* to be the last course attained by construction according to the preceding rule; draw the axis *o*B, and produce it and *fd* to meet in P; from the centre P, with the distance P*d*, describe the arc *dl*; join *dl*, and produce the line to *m* on the curve AB; at *m*, draw *mf*, *mh*, and mark the points of their intersection *of oB by tu; draw fg* and *hr* parallel to AC; then will *ft* be half

the length, *lt* the height or versed sine of the first course, and *hr* the half length, and *ru* the height or versed sine of the remaining course.

Note.—The height or versed sine corresponding to a given length of any of these courses, is found as already shewn.

To cover a dome by the second method.

Let ABC, fig. 44, be the section of a dome; then the length of a course of covering is obtained as follows: the length of the course C*e* is equal to the curve AC, and *kf*, the breadth of it; join *fh*, and the lines 1, 2, 3, and 4 intersected thereby will be the half-breadths (for the vertical C*h*) of the course at the corresponding lines C, *e*, through which points a line can be drawn, which will give the form of the course required, as *ekf*, as in the following.

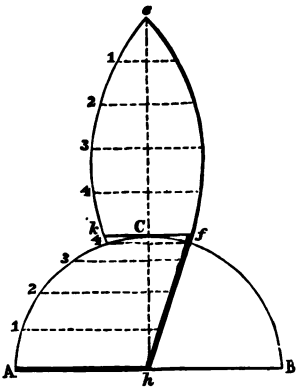


Fig. 44.

To ascertain the outline of a course of covering to a dome, without reference to a section of the dome.

Let AB be the breadth of the course; bisect it at *c* by the perpendicular *df*; make *cf* equal to the length of the arc from the base of the dome to the top of it (which may be found either by measurement or calculation); divide the semicircle A*d*B into any number of equal parts, and draw the sines parallel to AB; divide *cf* into the same number of equal parts, and draw lines parallel to AB; make ordinates on each side of *cf*, as 1, 2, and 3, equal to the sines of A*d*B, and a curve drawn through their terminations 1, 2, and 3, on both sides, will give the outline of the course.

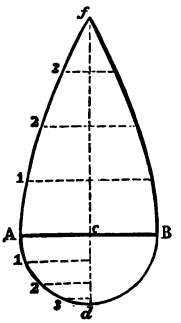


Fig. 45.

To ascertain by calculation the breadths of the courses of covering to a dome when in vertical sections, and of a given or required width.

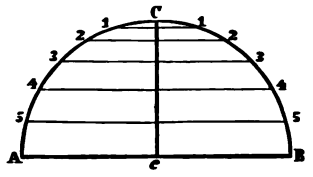


Fig. 46.

Let ABC be the vertical section of a dome, and the required width of the courses to be $\frac{1}{24}$ of the circumference of the base. Bisect AB in *e*, and erect a perpendicular *e*C; divide the arc BC into a number of equal parts (six), and from the points 1, 2, 3, &c., draw lines parallel to AB; and as the breadth of the courses at AB is to be $\frac{1}{24}$ of the circumference thereof, it follows that the breadth at 1 1, 2 2, &c., will also be $\frac{1}{24}$ of the circumference at these points; the diameter then being ascertained at these points, the operation is as follows:—

Assume AB to be 12 feet in diameter, and that—

5 5	is by	measurement	11·6
4 4	„	„	10·4
3 3	„	„	8·5
2 2	„	„	6·
And 1 1	„	„	3·1

Then the circumference at AB will be 12 × 3·1416, or 37·6992; hence the breadth of a course at that line will be 37·6992 ÷ 24, or 1·57 feet. The circumference at 5 5 will be 11·6 × 3·146, or 36·446, and the breadth of the course at the line will be 36·446 ÷ 24, or 1·52 feet.

In like manner, the circumference at 4 4, 3 3, 2 2, 1 1, will be 32·673, 26·704, 18·850, and 9·739, and the breadth of the courses respectively, 1·36 and 1·11 feet, ·785 and ·405 inches.

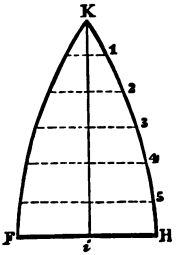


Fig. 47.

Draw FH equal to $\frac{1}{24}$ of the circumference of AB, fig. 47; bisect it at *i* by a perpendicular line *i*K, and set off thereon six equal distances, 5, 4, 3, 2, and 1, the same as upon the dome, and make the breadths at these points equal to those found by calculation; then a line drawn through the points will give the outside of the course required.

Application of the preceding Rules to the Shells of Cylindrical Boilers and Flues.

The courses of a cylindrical boiler or flue, when tapered, are a series of conic frustums inserted into each other, the height of each being the breadth of the plate of the course; the diameters at the large and small ends being respectively equal to those of the external and internal diameters of the boiler or flue.

To ascertain how to cut a template for the tapering courses of a boiler or flue.

Suppose it is required to build a cylindrical boiler 5 feet in diameter, the shell of which is to be made in four plates of equal length, of a breadth of 22 inches, $\frac{3}{8}$ of an inch thick.

Note.—In referring to the length or breadth of a boiler, or any riveted plate, the measurements are taken and given from the centre of the rivet holes.

The first operation is to find the vertex of the cone of which the course is a frustum, the rule of which is as follows:—

Multiply the breadth of the course by half the diameter of the boiler; divide their product by the thickness of the metal, and the quotient will be the length of the slant edge of the course.

Inches.

Thus, as $\frac{3}{8}$ (·375) : 30 :: 22 : slant edge of cone

30

·375)660

1760 inches.

Now, as the radius with which the template is to be drawn is of an inconvenient or impracticable length, the best method to find the curve is to find the versed sine corresponding to the curve, for which proceed as follows.

In order to delineate the necessary diagram, fig. 48, first obtain the following elements; viz., the radius EB, the distances BD *and AB, and which* are thus found:—

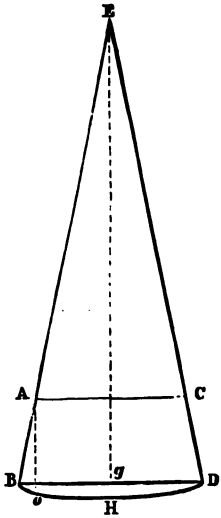


Fig. 48.

EB, as above shewn, is 1760 inches, BD is the fourth of the circumference of the shell, which is to consist of four plates like to AC, DB; consequently B*g* is one-half of one-fourth of the circumference of the shell; thus, the diameter of the boilers being 5 feet, the circumference is 188·5 inches; then, 188·5÷4 = 47·12, and 47·12 ÷2 = 23·56 inches; A*o* being equal to the width of the course, it is, as here given, 22 inches. EB is the radius, BD the chord, and *g*H the versed sine of the arc first required. The rule to obtain which is, from the square of the radius (EB) subtract the square of half the chord of the arc (*g*B), and the square root of the remainder subtracted from the radius will give the versed sine (*g*H).

ILLUSTRATION.

The radius is 1760, and half the chord of the arc is 23·56 inches.

Then	1760	23·56
	1760	23·56
	3097600 = EB²	555·0736 = *g*B²
	555·0736	

3097044·9264, the square root of which is 1759·84 = E*g*, and 1759·84 subtracted from 1760 = ·16 = *g*H the versed sine.

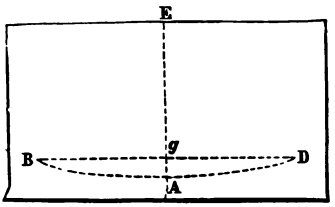


Fig. 49.

Draw upon a plate, fig. 49 (from which a template is to be made), a straight line, as BD = 47·12 inches; bisect it by AE, and set off *g*A, the versed sine, = ·16 inch; then, by bending a batten around the points B, A, D, the curve of the outside edge of the course may be readily drawn.

As the diameter of the small end of the required frustum is twice the thickness of the plate less than the larger, or $(60 - \frac{3}{8} \times 2) = 59\frac{1}{4}$ inches, the circumference will be $59\frac{1}{4} \times 3{\cdot}1416 = 186{\cdot}14$; hence the length of the plate on the inside will be $186{\cdot}14 \div 4 = 46{\cdot}53$ inches. The distance between the rivet holes is to be about 2 inches, consequently the templates will contain 23 divisions in the length and 11 in the breadth. The template can now be drawn, and will be as shewn in fig. 50—viz., $ab = 46{\cdot}53$ inches, $cd = 47{\cdot}12$ inches, and $ei = 22$ inches.

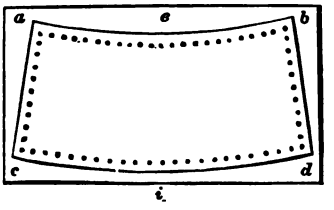


Fig. 50.

When the flue is of the form of fig. 50.

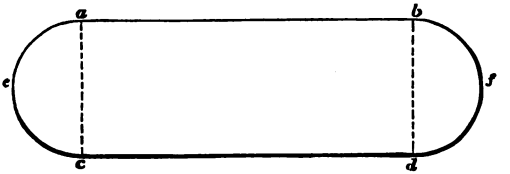


Fig. 51.

Assume *ab*, *cd*, each = 12 inches in length, $ac = 9$ inches in diameter, and that a course is to be 18 inches wide, and $\frac{1}{4}$ of an inch in thickness.

Then as $\frac{1}{4}$ in. : $4\frac{1}{2}$: : 18 : slant of edge of cone, or radius of circle of the outer edge of the template.

$$\begin{array}{r} \frac{1}{4} : 4\frac{1}{2} \; ; : 18 \\ 4\frac{1}{2} \\ \hline 72 \\ 9 \\ \hline \frac{1}{4})81 \\ \hline 324 \text{ inches.} \end{array}$$

The diameter of the semicircular ends being 9 inches, the circumference of each part will be $9 \times 3{\cdot}1416 \div 2 = 28{\cdot}2744 \div 2 = 14{\cdot}13$ *inches for the larger end,* and for the versed sine proceed as before

shewn. 324 being the radius of the arc, and $14{\cdot}13 \div 2 = 7{\cdot}06$ half the chord of the arc; therefore—

324	7·06
324	7·06
$104976 = ec^2$	$49{\cdot}8436 = ci^2$

and $104976 - 49{\cdot}8436 = 104926{\cdot}2$, the square root of which is 323·9, which, when subtracted from 324, gives $\cdot 1 = fi$, the versed sine of arc *cfd* of outer edge of template. (See figure 52.)

The diameter of the small side of the semicircular ends will be $(9 - \frac{1}{4} \times 2) = 8\frac{1}{2}$ in.; consequently the circumference of each will be

$$\frac{8\frac{1}{2} \times 3{\cdot}1416}{2} = \frac{26{\cdot}70}{2} = 13{\cdot}35 \text{ inches.}$$

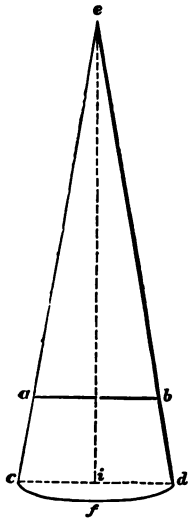


Fig. 52.

Note.—In these examples the author has endeavoured to point out that the length of the chord is measured upon the plate from the length of the curve of the arc; this, he admits, is not strictly correct, as the chord is shorter than the arc; this difference, however, is so little in all ordinary cases, that it was considered an unnecessary element to bring into the calculation for ordinary practice, inasmuch as it involves a complexity at variance with the author's purpose. In cases, however, requiring *exactness* of dimensions, the following manner of proceeding will be found exact:—

To find the chord of the arc.

Multiply the versed sine by 2, and subtract the product from the diameter; then subtract the square of the remainder from the square of the diameter, and the square root of the remainder will give the *chord required.*

$$\text{Or } \sqrt{\{d^2 - (d - \overline{v \times 2})^2\}} = \text{C.}$$

PART IV.—VELOCITIES IN BORING, AND WHEEL GEARING.

VELOCITIES.

Rule to find the velocity in boring cast iron.—Divide 25 by the diameter of the cylinder in inches, for the number of revolutions or part of a revolution per minute.

Notice.—The velocity required in boring and turning cast iron, wrought iron, and brass, with fixed cutters. In some cases the lathe mandril will make about 100 revolutions while the cutter goes over one inch in a lateral direction, and in others about 16 revolutions in going over the same distance; so that the velocity for cutting will always depend to a great extent on the feed given to the cutter, the strength of the lathe or machine, and on light or heavy work to be bored or turned; besides, the same kind of metals vary so much that this question, to a great extent, must always be left to the workman's own judgment.

Again, workmen differ so much as to what *is* the actual average velocity. The following table is given to shew what is generally acknowledged to be *the* average velocity for cast iron, wrought iron, and brass.

Table of Velocities in Boring and Turning

Cast Iron.			Malleable Iron.	
Inches Diameter.	Revolutions per minute in horizontal boring.	Turning revolutions per minute.	Inches Diameter.	Turning revolutions per minute.
1	25·	50·	1	60·
2	12·5	25·	2	30·
3	8·33	16·66	3	20·
4	6·25	12·5	4	15·
5	5·	10·	5	12·
6	4·16	8·32	6	10·
7	3·57	7·14	7	8·5
8	3·12	6·24	8	7·5
9	2·77	5·54	9	6·5
10	2·5	5·	10	6·
12	2·	4·	12	5·
15	1·66	3·32	15	4·
20	1·25	2·5	20	3·
25	1·	2·	25	2·4
30	·83	1·66	30	2·
35	·71	1·42	35	1·7
40	·625	1·25	40	1·5
50	·5	1·	50	1·2
60	·417	·834	60	1·
70	·358	·716	70	·85
80	·313	·626	80	·75
90	·278	·556	90	·66
100	·25	·5	100	·6

N.B.—For vertical boring use the velocities given for turning.

Velocity required in Boring Cast Iron and Brass with Fixed Cutters.

In boring a cast-iron cylinder 25 inches in diameter, the boring bar requires to make one revolution per minute, at which speed the cutters will move through about 78 inches in the circumference. In boring brass the speed is much slower than that of cast iron. The bar requires to make one revolution per minute in boring 12 inches diameter, which is equal to a velocity in the cutters of about 38 inches in the circumference. If this velocity *be materially* exceeded the cutters will be destroyed, and the *cylinder* or pump made taper.

Turning has a velocity double that of boring with cutters. In turning malleable iron the velocity is quicker than either cast iron or brass; for, in turning a shaft 6 inches in diameter it will require to make 10 revolutions per minute, which is equal to a speed on the tool of over 15 feet.

Examples.

(1.) Required the velocity to bore a cast-iron cylinder 75 inches in diameter. $\frac{25}{75} = \frac{1}{3}$ of revolution, or 1 revolution in three minutes.

Rule for turning cast iron.—Divide 50 by the diameter for the number of revolutions or part of a revolution per minute.

Notice in the preceding table of velocities for boring cast iron, where the velocity for 100 inches is ·25, or $\frac{25}{100}$, equal, when reduced, to $\frac{1}{4}$ of a revolution, or 1 revolution in four minutes, that the numerator gives the revolutions and the denominator the time in minutes.

Rule for boring brass.—Divide 12 by the diameter in inches for the number of revolutions or part of a revolution per minute.

(2.) Required the velocity to bore a brass air-pump 24 inches diameter, $\frac{12}{24} = ·5$, or 1 revolution in two minutes.

Rule for turning malleable iron.—Divide 60 by the diameter in inches for the number of revolutions or part of a revolution per minute.

To compute the weight of cast metal by the weight of the pattern, when the same is of white pine.

Rule.—Multiply the weight of the pattern in pounds by the following multipliers, and the product will give the weight of the casting :—

Iron, 14; *Brass*, 15; *Lead*, 22; *Tin*, 14; *Zinc*, 13·5.

When there are circular cores or prints—Multiply the square of the diameter of the core or print by its length in inches, the product by ·0175, and the result is the weight of the pattern of the core or print to be deducted from the weight of the pattern. It is customary in the making of patterns for castings to allow for contraction per lineal foot of pattern, as follows :—*Iron and Lead*, $\frac{1}{8}$ of an inch per foot; *Brass and Zinc*, $\frac{3}{16}$ per foot; and *Tin* $\frac{1}{12}$ per foot.

WHEEL GEARING.

Definitions.

The *Pitch Line of a Circular Wheel* is the circle upon which the pitch is measured, and it is the circumference by which the diameter or the velocity of the wheel is measured.

The *Pitch* is the arc of the circle of the "pitch line," and is determined by the number of teeth in the wheel.

The *True Pitch* (Chordial), or that by which the dimensions of the tooth of a wheel alone are determined, is a line drawn from the centre of one tooth to the centre of the next upon the pitch line.

The *Line of Centres* is the line between the centres of two wheels.

The *Radius of a Wheel* is the semi-diameter running to the periphery of a tooth.

The *Pitch Radius* is the semi-diameter running to the pitch line.

The *Length of a Tooth* is the distance from its base to its extremity—*i.e.*, from root to crest.

A *Cog-wheel* is the general term for a wheel having a number of cogs or teeth set upon or radiating from its circumference.

A *Mortise Wheel* is a wheel constructed for the reception of teeth or cogs, which are fitted into recesses or sockets upon the face of the wheel.

A *Rack* is a series of teeth set in a plane.

A *Sector* is a wheel which reciprocates without forming a full revolution.

A *Spur-wheel* is a wheel having its teeth perpendicular to its axis.

A *Bevel-wheel* is a wheel having its teeth at an angle with its axis.

A *Mitre-wheel* is a wheel having its teeth at an angle of 45° with its axis.

A *Face-wheel* is a wheel having its teeth set upon one of its sides.

An *Annular* or *Internal Wheel* is a wheel having its teeth convergent to its centre.

Spur Gear wheels act upon each other in the same plane.

Bevel Gear wheels act upon each other at an angle.

When the tooth of a wheel is made of material different from that of the wheel, it is termed *a cog*, in a *Pinion* it is termed a leaf, and in a *trundle* a *stave*.

A wheel which impels another is termed the *Spur*, *Driver*, or *Leader*; the one impelled is the *Pinion*, *Driven*, or *Follower*.

A series of wheels in connection with each other is termed a *Train*.

When two wheels act upon each other, the greater is termed *the Wheel,* and the *lesser* the *Pinion.*

EXPLANATIONS.

To *increase* or *diminish* velocity in a given proportion (with the least quantity of wheel-work), the number of teeth in each pinion should be to the number of teeth in its wheel, *as* 1 *is to* 3·59; even to save *space* and *expense,* the ratio should *never exceed* as 1 is to 6.

The least number of teeth that it is practicable to give to a wheel is regulated by the necessity of having at least *one pair* (or two teeth) always in gear (or action), in order to provide for the contingency of a tooth breaking.

The number of teeth in a wheel should *always* be *prime* to the number in the pinion—that is, the number of teeth in the wheel should not be divisible by the number of teeth in the pinion without a remainder. This is in order to prevent the same teeth *coming together* so often as to cause an *irregular wear* of their faces. An *odd tooth* introduced into a wheel is termed a *hunting-tooth* or *cog.*

To compute the pitch of a wheel.

Rule.—Divide the circumference at the pitch-line by the number of teeth.

EXAMPLE.

A wheel 40 inches in diameter requires 75 teeth, what is the pitch?

$$\frac{3{\cdot}1416 \times 40}{75} = 1{\cdot}675 \text{ inches.}$$

Another Method.

Rule.—Divide 180° by the number of teeth, ascertain the sine of the quotient, and multiply it by the diameter of the wheel.

EXAMPLE.

The number of teeth is 75, and the diameter 40 inches, what is the true pitch?

$\frac{180}{75} = 2°\ 24'$ and sine of $2°\ 24' = {\cdot}04188$, which multiplied by *40 equals 1·675 inches.*

To compute the diameter of a wheel when the pitch is given.

Rule.—Multiply the number of teeth by the pitch, and divide the product by 3·1416.

EXAMPLE.

The number of teeth in a wheel is 75, and the pitch 1″·675 inches, what is the diameter?

$$\frac{75 \times 1{\cdot}6755}{3{\cdot}1416} = 40 \text{ inches.}$$

Another Method.

Rule.—Multiply the number of teeth in the wheel by the true pitch, and again by ·3184.

EXAMPLE.

Take the preceding case: $7 \times 51{\cdot}6752 \times {\cdot}3184 = 40$ inches.

To compute the number of teeth in a wheel.

Rule.—Divide the circumference by the pitch.

To compute the circumference of a wheel.

Rule.—Multiply the number of teeth by the pitch.

To compute the number of teeth in a pinion to have a given velocity.

Rule.—Multiply the velocity of the driver by its number of teeth, and divide the product by the velocity of the driven.

EXAMPLES.

(1.) The velocity of a driver is 16 revolutions, the number of teeth is 54, and the velocity of the pinion is 48, what is the number of its teeth?

$$\frac{16 \times 54}{48} = 18 \text{ teeth.}$$

(2.) A wheel having 75 teeth is making 16 revolutions per minute, what is the number of teeth required in the pinion to make 24 revolutions in the same time?

$$\frac{16 \times 75}{24} = 50 \text{ teeth.}$$

To compute the diameter of a pinion when the diameter of the wheel is given.

Rule.—Multiply the diameter of the wheel by the number of teeth in the pinion, and divide the product by the number of teeth in the wheel.

EXAMPLE.

The diameter of a wheel is 25 inches, the number of its teeth 210, and the number of teeth in the pinion 30, what is the diameter of the pinion?

$$\frac{25 \times 30}{210} = 3{\cdot}57 \text{ inches.}$$

To compute the true diameter (at the pitch line) of a wheel when the true or chordial pitch is given.

Divide 180 by the number of teeth required, afterwards ascertain the cosecant of the quotient, and multiply it by the pitch, which will give the true diameter.

EXAMPLE 1. A wheel of 12 teeth and ·608 pitch.

$$\frac{180}{12} = 15°$$

Cosec. 15° = 3·8637033
Pitch, ·608

309096264
231822198

2·34 nearly.

EXAMPLE 2. A wheel of 62 teeth and ·35 pitch.

$$\frac{180°}{62} = 2°54' \text{ (nearly).}$$

The cosecant of 2 degrees and 54 minutes = 19·765604, which multiplied by the pitch equals the correct diameter at the pitch line.

Cosecant 19·765604
Pitch ·35

98828020
59296812

6·91796140·

Diameter 6·918 nearly (3 decimal figures being sufficient).

TABLE TO FIND THE DIAMETERS OF TOOTHED WHEELS, THE RADIUS OF A WHEEL WHEN THE PITCH IS GIVEN, OR THE PITCH OF A WHEEL WHEN THE RADIUS IS GIVEN, THAT SHALL CONTAIN FROM 10 TO 150 TEETH, AND ANY PITCH WHATEVER.

Number of Teeth.	Radius.	Number of Teeth.	Radius.	Number of Teeth.	Radius.	Number of Teeth.	Radius.
10	1·618	46	7·327	82	13·051	118	18·782
11	1·775	47	7·486	83	13·210	119	18·941
12	1·932	48	7·645	84	13·370	120	19·101
13	2·089	49	7·804	85	13·531	121	19·260
14	2·247	50	7·963	86	13·691	122	19·419
15	2·405	51	8·122	87	13·849	123	19·578
16	2·563	52	8·281	88	14·008	124	19·737
17	2·721	53	8·440	89	14·168	125	19·896
18	2·879	54	8·599	90	14·327	126	20·055
19	3·038	55	8·758	91	14·486	127	20·214
20	3·196	56	8·917	92	14·645	128	20·374
21	3·355	57	9·076	93	14·804	129	20·533
22	3·513	58	9·235	94	14·963	130	20·692
23	3·672	59	9·395	95	15·122	131	20·851
24	3·831	60	9·554	96	15·281	132	21·010
25	3·989	61	9·713	97	15·440	133	21·169
26	4·148	62	9·872	98	15·600	134	21·328
27	4·307	63	10·031	99	15·759	135	21·488
28	4·466	64	10·190	100	15·918	136	21·647
29	4·625	65	10·349	101	16·077	137	21·806
30	4·783	66	10·508	102	16·236	138	21·965
31	4·942	67	10·667	103	16·395	139	22·124
32	5·101	68	10·826	104	16·554	140	22·283
33	5·260	69	10·985	105	16·713	141	22·442
34	5·419	70	11·144	106	16·873	142	22·602
35	5·578	71	11·303	107	17·032	143	22·761
36	5·735	72	11·463	108	17·191	144	22·920
37	5·896	73	11·622	109	17·350	145	23·079
38	6·055	74	11·781	110	17·509	146	23·238
39	6·214	75	11·940	111	17·668	147	23·397
40	6·373	76	12·099	112	17·827	148	23·556
41	6·532	77	12·258	113	17·987	149	23·716
42	6·691	78	12·417	114	18·146	150	23·875
43	6·850	79	12·576	115	18·305		
44	7·009	80	12·735	116	18·464		
45	7·168	81	12·893	117	18·623		

Rule.—Multiply the radius in the table by the pitch given, and the product will be the radius of the wheel required; or divide the *radius of the wheel* by the radius in the table, and the quotient *will be the pitch of* the wheel required.

Examples.

(1.) Required the radius of a wheel to contain 64 teeth, 3 inches pitch. Here, $10{\cdot}19 \times 3 = 30{\cdot}57$ inches.

(2.) What is the diameter of a wheel to contain 62 teeth, ·35 pitch? Here,

$$\begin{array}{r} 9{\cdot}872 \\ {\cdot}35 \\ \hline 49360 \\ 29616 \\ \hline 3{\cdot}45520 \\ \times\ 2 \\ \hline 6{\cdot}91040 \end{array}$$

THE PITCH OF WHEELS.

We have seen that the pitch of a wheel is the length of an arc of the pitch line, comprising a tooth and space. Millwrights ordinarily measure the pitch as a chord of this arc, and except in pinions with very few teeth, the two measurements sensibly coincide.

Having the diameter of a wheel and the number of teeth, the pitch may be found as follows:—

Let D be the diameter of a wheel, N the number of teeth, and p the pitch; then, as $3{\cdot}1416\ \mathrm{D}$ = the circumference of the circle,

$$p = \frac{3{\cdot}1416\ \mathrm{D}}{\mathrm{N}}$$

or approximately—

$$= \frac{22\ \mathrm{D}}{7\ \mathrm{N}}$$

Conversely, if the pitch of a wheel be given and the number of teeth, then the diameter may be found—

$$D = \frac{p\mathrm{N}}{3{\cdot}1416} = \frac{7\ \mathrm{N}p}{22} \text{ nearly.}$$

And if the pitch and diameter of a wheel be given, then the number of teeth may be found—

$$N = \frac{3{\cdot}1416}{p} = \frac{22D}{7p} \text{ nearly.}$$

But since a wheel must contain a whole number of teeth, N may never be a mixed number. If, therefore, this equation gives N with a fraction, a wheel cannot be constructed of that diameter and pitch. In this case, however, by slightly increasing or decreasing either the diameter or the pitch, the necessary conditions may be complied with.

In practice it is convenient to limit the number of pitches, and the following series gives all the most ordinary pitches of my own practice:—

Spur fly wheels, 5, $4\frac{1}{2}$, 4, $3\frac{1}{2}$, $3\frac{1}{4}$, 3, $2\frac{1}{2}$, 2, $1\frac{1}{2}$ inches.

Spur and bevel wheels, 5, $4\frac{1}{2}$, 4, $3\frac{1}{2}$, $3\frac{1}{4}$, 3, $2\frac{3}{4}$, $2\frac{1}{2}$, $2\frac{1}{4}$, $2\frac{1}{8}$, 2, $1\frac{3}{4}$, $1\frac{5}{8}$, $1\frac{1}{2}$, $1\frac{3}{8}$, $1\frac{1}{4}$, $1\frac{1}{8}$, 1, $\frac{7}{8}$ inches. Wheels of smaller pitch than this are not used in millwork; but in machines, &c., the following pitches would probably be sufficient, viz.:—

$$1, \tfrac{3}{4}, \tfrac{5}{8}, \tfrac{1}{2}, \tfrac{3}{8}, \tfrac{1}{4} \text{ inch.}$$

The value of $\pi = \frac{22}{7}$ ordinarily employed is not very accurate; hence it is convenient to calculate beforehand the values of $\frac{p}{3{\cdot}1416}$ and $\frac{3{\cdot}1416}{p}$ for the most useful pitches.

The following table gives these values:—

Pitch in Inches.	$\frac{3{\cdot}1416}{\text{Pitch.}}$	$\frac{\text{Pitch.}}{3{\cdot}1416}$	Pitch in Inches.	$\frac{3{\cdot}1416}{\text{Pitch.}}$	$\frac{\text{Pitch.}}{3{\cdot}1416}$
5	0·6283	1·5915	$1\frac{3}{4}$	1·7952	0·5570
$4\frac{1}{2}$	0·6981	1·4270	$1\frac{5}{8}$	1·9264	0·5141
4	0·7854	1·2732	$1\frac{1}{2}$	2·0944	0·4774
$3\frac{1}{2}$	0·8976	1·1141	$1\frac{3}{8}$	2·2848	0·4377
$3\frac{1}{4}$	0·9666	1·0345	$1\frac{1}{4}$	2·5132	0·3978
3	1·0472	0·9548	$1\frac{1}{8}$	2·7924	0·3580
$2\frac{3}{4}$	1·1333	0·8754	1·	3·1416	0·3182
$2\frac{1}{2}$	1·2566	0·7958	$\frac{7}{8}$	3·5904	0·2785
$2\frac{1}{4}$	1·3963	0·7135	$\frac{3}{4}$	4·1888	0·2386
2	1·5708	0·6366	$\frac{5}{8}$	5·0265	0·1988
$1\frac{7}{8}$	1·6755	0·5937	$\frac{1}{2}$	6·2832	0·1591

Rule.—1. Given the pitch and number of teeth in a wheel to find its diameter.

Multiply the number of teeth by the constant in the third or sixth column of the preceding table corresponding to the pitch.

2. Given the pitch and diameter of a wheel to find the number of teeth.

Multiply the diameter by the constant in the second or fifth column of the table corresponding to the pitch.

If this rule gives a mixed number, or whole number and fraction, a wheel cannot be constructed, as before said. The most convenient way of proceeding in that case will be to take the nearest whole number to the number given by the rule, and, using Rule 1, find a new diameter, which will differ but slightly from the one previously assumed. This new diameter must be taken for the pitch circle in constructing the wheel.

Thus, suppose it required to find the diameter of a wheel of 2 inches pitch and 150 teeth. By Rule 1, we have $D = 150 \times 0{\cdot}6366 = 95\frac{1}{2}$ inches $= 7$ feet $11\frac{1}{2}$ inches.

Or, required the number of teeth in a wheel of 3 inches pitch and 9 feet diameter. By Rule 2 $N = 108 \times 1{\cdot}0472 = 113{\cdot}097$. Here the wheel will contain very nearly 113 teeth; but if we wish to know more accurately the diameter of a wheel of 3 inches pitch and 113 teeth we find by Rule 1, $D = 113 \times 0{\cdot}9548 = 107{\cdot}89$ inches $= 8$ feet $11\frac{9}{10}$ inches. That is, a wheel of exactly 9 feet could not be constructed with a 3-inch pitch, but one of 8 feet $11\frac{9}{10}$ inches might be constructed and would contain 113 teeth.

Professor Willis has employed another method of graduating the sizes of wheels. Suppose the diameter, instead of the circumference, to be divided into as many equal parts as the wheel has teeth, and let one of these parts be called the diametral pitch of the wheel, to distinguish it from the common or circular pitch. Let M be the diametral pitch, so that

$$\frac{D}{N} = M,$$

and let a series of values be taken for M in simple fractions of an inch, so that

$$M = \frac{1}{m},$$

where N and m are always whole numbers.

The ordinary values of m are 20, 16, 14, 12, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, which include wheels in which the circular or common pitch varies from $\frac{1}{8}$ inch to 3 inches, as shewn in the following table given by Professor Willis:—

Value of m.	Circular pitch in inches and decimals.	Circular pitch to nearest one-sixteenth.	Value of m.	Circular pitch in inches and decimals.	Circular pitch to nearest one-sixteenth.
3	1·047	1	9	·349	—
4	·785	$\frac{3}{4}$	10	·314	$\frac{5}{16}$
5	·628	$\frac{5}{8}$	12	·262	$\frac{1}{4}$
6	·524	$\frac{1}{2}$	14	·224	—
7	·449	$\frac{7}{16}$	16	·196	$\frac{3}{16}$
8	·393	$\frac{3}{8}$	20	·157	$\frac{1}{8}$

This system is convenient where wheels of small pitch are employed, and involves less calculation than the common system.

Since $\frac{D}{N} = M$, we have $M = \frac{p}{3{\cdot}1416}$. Therefore, in the previous table (page 106) the quantities in the third and sixth columns are the diametral pitches corresponding to the circular pitches in the first column, and the numbers in the second column are the corresponding values of m. In fact, this scheme differs from the first, simply by expressing in small whole numbers the quantity $\frac{3{\cdot}1416}{p}$ instead of p.

The following table gives the relation of diameter, pitch, and number of teeth for wheels of from $\frac{1}{2}$ inch to 5 inches pitch, and of from 20 to 200 teeth. Intermediate numbers may be found by direct proportion, by multiplying the number given for a wheel of half or a third of the number of teeth by 2 or 3, or by adding together the diameters given for 2 wheels, the sum of whose teeth is the number required. For an odd number of teeth add the number given at the head of the table as many times as may be *necessary, to the* diameter for a wheel of the nearest number of *teeth given.*

TABLE SHEWING THE RELATION OF PITCH, DIAMETER, AND NUMBER OF TEETH.

Number of Teeth in Wheel.	Pitch in Inches.															
	¼	⅜	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	4	4½	5
For each tooth add	·1591	·2386	·3182	·3978	·4774	·5570	·6366	·7135	·7958	·8754	·9548	1·035	1·114	1·273	1·427	1·592
12	1·91	2·86	3·82	4·77	5·73	6·68	7·64	8·56	9·55	10·50	11·46	12·41	13·37	15·28	17·12	19·10
13	2·07	3·10	4·14	5·17	6·21	7·24	8·28	9·28	10·35	11·38	12·41	13·45	14·48	16·55	18·55	20·69
14	2·23	3·34	4·46	5·57	6·68	7·80	8·91	9·99	11·14	12·26	13·37	14·48	15·60	17·83	19·98	22·28
15	2·39	3·58	4·77	5·97	7·16	8·36	9·55	10·70	11·94	13·13	14·32	15·52	16·71	19·10	21·41	23·87
16	2·55	3·82	5·09	6·37	7·64	8·91	10·19	11·41	12·73	14·01	15·28	16·55	17·83	20·37	22·83	25·46
17	2·70	4·06	5·41	6·77	8·12	9·47	10·82	12·13	13·53	14·88	16·23	17·59	18·94	21·64	24·26	27·06
18	2·86	4·30	5·73	7·17	8·60	10·03	11·46	12·84	14·32	15·76	17·19	18·62	20·05	22·92	25·69	28·65
19	3·02	4·54	6·05	7·56	9·08	10·58	12·10	13·56	15·12	16·63	18·14	19·66	21·17	24·19	27·11	30·24
20	3·18	4·77	6·36	7·96	9·55	11·14	12·73	14·27	15·92	17·51	19·10	20·69	22·28	25·46	28·54	31·83
21	3·34	5·01	6·68	8·36	10·02	11·70	13·37	14·98	16·71	18·38	20·05	21·72	23·40	26·74	29·97	33·42
22	3·50	5·25	7·00	8·76	10·50	12·25	14·01	15·70	17·51	19·26	21·01	22·76	24·51	28·01	31·39	35·01
23	3·66	5·49	7·32	9·16	10·98	12·81	14·64	16·41	18·30	20·13	21·96	23·79	25·62	29·28	32·82	36·60
24	3·82	5·73	7·64	9·55	11·46	13·37	15·28	17·12	18·10	21·01	22·92	24·83	26·74	30·56	34·25	38·20
25	3·97	5·97	7·96	9·96	11·94	13·93	15·92	17·84	19·90	21·89	23·87	25·86	27·85	31·84	35·68	39·79
30	4·77	7·16	9·55	11·93	14·32	16·71	19·10	21·41	23·87	26·26	28·64	31·04	33·42	38·21	42·81	47·74
35	5·57	8·35	11·14	13·92	16·71	19·50	22·28	24·97	27·85	30·64	33·42	36·21	38·99	44·57	49·95	55·70
40	6·36	9·54	12·73	15·91	19·10	22·28	25·46	28·54	31·83	35·02	38·19	41·38	44·56	50·94	57·08	63·66
45	7·16	10·74	14·32	17·90	21·49	25·07	28·65	32·11	35·81	39·39	42·97	46·55	50·13	57·30	64·22	71·62
50	7·96	11·93	15·91	19·89	23·87	27·85	31·83	35·67	39·79	43·77	47·74	51·73	55·71	63·67	71·35	79·58
55	8·75	13·12	17·50	21·88	26·26	30·64	35·01	39·24	43·77	48·15	52·51	56·90	61·28	70·04	78·49	87·53
60	9·55	14·32	19·09	23·87	28·64	33·42	38·20	42·81	47·75	52·52	57·29	62·07	66·85	76·40	85·62	95·49
65	10·34	15·51	20·68	25·86	31·03	36·21	41·38	46·38	51·72	56·90	62·06	67·24	72·42	82·77	92·76	103·45
70	11·14	16·70	22·27	27·85	33·42	38·99	44·56	49·94	55·71	61·28	66·84	72·42	77·99	89·13	99·89	111·41
75	11·93	17·89	23·86	29·84	35·81	41·78	47·75	53·51	59·69	65·66	71·61	77·59	83·56	95·50	107·03	119·36
80	12·73	19·09	25·46	31·82	38·19	44·56	50·93	57·08	63·66	70·03	76·38	82·76	89·13	101·87	114·16	127·32
85	13·52	20·28	27·05	33·81	40·58	47·35	54·11	60·65	67·64	74·41	81·16	87·93	94·70	108·23	121·30	135·28
90	14·32	21·47	28·64	35·80	42·97	50·13	57·29	64·21	71·62	78·78	85·93	93·11	100·27	114·60	128·43	143·25
95	15·11	22·66	30·23	37·79	45·36	52·91	60·47	67·78	75·60	83·16	90·70	98·28	105·84	120·96	135·57	151·19
100	15·91	23·86	31·82	39·78	47·74	55·70	63·66	71·35	79·58	87·54	95·48	103·45	111·41	127·32	142·70	159·15
110	17·50	26·24	35·00	43·76	52·51	61·27	70·03	78·48	87·54	96·29	105·03	113·80	122·55	140·05	156·97	175·07
120	19·09	28·63	38·18	47·74	57·28	66·84	76·39	85·62	95·50	105·05	114·58	124·14	133·69	152·78	171·24	190·98
130	20·68	31·02	41·36	51·72	62·06	72·41	82·76	92·75	103·45	113·80	124·12	134·50	144·83	165·52	185·51	206·90
140	22·27	33·40	44·54	55·70	66·84	77·98	89·12	99·89	111·41	122·56	133·67	144·83	155·97	178·25	199·78	222·81
150	23·86	35·79	47·73	59·67	71·61	83·55	95·49	107·03	119·37	131·31	143·22	155·18	167·12	190·98	214·05	238·73
160	25·45	38·18	50·91	63·65	76·38	89·12	101·86	114·16	127·33	140·06	152·77	165·52	178·26	203·71	228·32	254·64
170	27·04	40·56	54·10	67·63	81·16	94·69	108·22	121·99	135·29	148·82	162·32	175·87	189·40	216·44	242·59	270·56
180	28·64	42·95	57·28	71·60	85·93	100·26	114·59	128·43	143·24	157·57	171·86	186·21	200·54	229·18	256·86	286·47
190	30·23	45·33	60·46	75·58	90·71	105·83	120·95	135·57	151·20	166·33	181·41	196·56	211·68	241·91	271·13	302·39
200	31·82	47·72	63·64	79·56	95·48	111·40	127·32	142·70	159·16	175·08	190·96	206·90	222·82	254·64	285·40	318·30

CONSTRUCTION OF EPICYCLOIDAL TEETH.

The epicycloid is the curve traced by a fixed point in the circumference of a circle, which rolls over or within the circumfer*ence of another circle,* or on a straight line.

Since every tooth in a wheel is of precisely the same form, it is sufficient to construct a single pattern tooth of true epicycloidal curvature, which may be used in setting all the other teeth.

First method, when the generating circle is the same for a wheel and pinion, the face of the tooth an epicycloid, and the flank a hypocycloid.

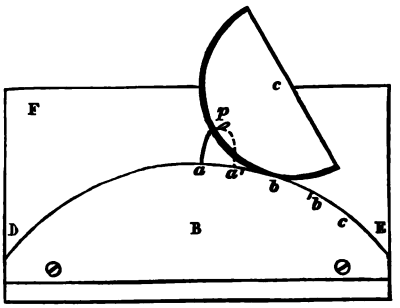


Fig. 53.

Construct two templets, A and B (figs. 54 and 55), having their faces arcs of the pitch circle of the wheel for which the tooth is required, and a third templet *c* cut to an arc of the intended generating circle of the epicycloid. Fix a steel tracing point *p* in the edge of the templet *c*, and for convenience a board F, on which to draw the tooth, may be fixed beneath the templet B. Mark off on the board (fig. 54) the pitch circle of the wheel DE, and take distances *ab*, *bc*, equal to the pitch of the teeth, and distances *aa'*, *bb'*, equal to the thickness of the teeth. If, then, the templet *c* be placed touching B, and with the tracing point *p* coinciding with one of the marks, as *a*, and be rolled toward E, the point will trace out an epicycloid *ap* on the board F, which will form one face of the tooth.

Next, let the point *p* be made to coincide *a'*, and the templet *c* be rolled toward D, the other face of the tooth will be described.

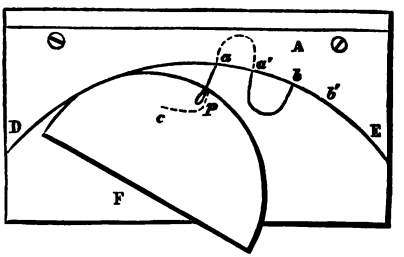


Fig. 54.

To draw the flanks, the templet A must now be fixed on the board F, with its face in contact with B; remove B and describe hypocycloids (fig. 55) from *a* and *a'*, by rolling *c* on the inside of the pitch circle.

This process, which, though complicated in description, is very easy in practice, must be repeated with two templets cut to the pitch circle of the pinion, the same *generating circle c* being employed; a similar pattern tooth will

thus be found for the pinion, which will work with that already found for the wheel. The usual custom in practice is for the millwright first to describe the hypocycloidal forms of the teeth required in the wheel and pinion; he then constructs two model teeth, one for the wheel and the other for the pinion, and from these he determines the true curves, and by means of his compasses transfers the same to the wheels or patterns on which these forms are to be impressed. The generating circle, it may be observed, must not exceed in size the radius of the pinion, or it would give rise to a weak form of tooth, thinner at the root than at the pitch circle.

Second method, where two generating circles are employed, in order that the flanks of the teeth may be straight lines radii of the wheel and pinion respectively.

It is the usual practice of millwrights to make the parts of the teeth of wheels within the pitch circles radii of the wheel. Now, if a hypocycloid be described by a generating circle equal in diameter to the radius of the wheel, and the face of the tooth of the pinion be described by a templet cut to a radius equal to half that of the wheel, and the flank of the tooth of the pinion and face of that of the wheel be described by a templet cut to a radius half that of the pinion, then these teeth will work together truly, and will have radial flanks.

Since it is unnecessary to describe the flanks of such teeth by templets, there will be needed only one templet cut to the pitch circle of each wheel, but templets of two generating circles are required. In other respects the method is identical with that already described. The great defect of this method is, that neither the wheel nor pinion will work accurately with a wheel or pinion of any other diameter than that for which they were originally made, and thus a vast number of wheel patterns must be made to fulfil the requirements of practice; whereas wheels described by the previous method will work equally well with all other wheels the teeth of which have been described by the same generating circle—it being understood that only the parts of teeth *without* the pitch circle of the wheel roll on the parts *within* the pitch circle of the pinion, and those without the pitch circle of the pinion on those within the pitch circle of the wheel.

Hence Professor Willis has been led to suggest that for a given set of wheels a constant generating circle should be taken to describe both the parts without and within the pitch circles of the whole series, instead of making that circle depend on the diameters of the wheels. In this case the first solution must be employed, and the flanks of the teeth will not be straight; but the great advantage is gained—that any pair of wheels in the series will work *together equally well.*

To determine the proper size of the generating circle, we must remember that a tooth of weak form is produced when the generating circle is greater than half the diameter of the wheel; hence the generating circle may be best made of a diameter equal to the radius of the smallest pinion of the series which are to work together.

The rack is the extreme case of a wheel, or may be considered as a wheel of infinite radius. It may be described by either of the preceding methods, only noting that, if the second method be employed, the generating circle which traces the face of the teeth of the wheel becomes a straight line, and the epicycloid becomes an involute.

If the teeth of a series of wheels and of a rack be described by the same generating circle, any of the wheels will work with equal accuracy into the rack.

INVOLUTE TEETH.

The Involute.

The curve traced by a flexible line unwinding from the circumference of a circle is called an involute.

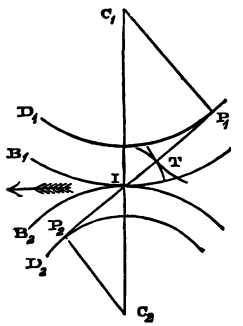


Fig. 55.

Let B_1 and B_2 (fig. 55) be the pitch lines of a wheel and pinion, and let C_1 and C_2 be their centres. From C_1 and C_2 describe two circles D_1, D_2, with radii C_1P_1 and C_2P_2 of the wheel and pinion respectively; so that

$$C_1I \ : \ C_2I \ :: \ C_1P_1 \ : \ C_2P_2.$$

Let T and T be two involute curves described by flexible lines unrolling from the circles D_1 and D_2 respectively, and touching at T. Then, if TI, TP_1 be drawn tangents to the circles at the points P_1 and P_2, they are also in one straight line, because they are both normals to the curves at T. It may also be shewn that the line P_1P_2 intersects C_1C_2 in I, where the pitch lines touch. Hence we have found two curves such that the line perpendicular to their common tangent passes in all positions of the wheel and pinion through I, which is the sufficient condition of their uniform motion, if moved by the sliding of the curves instead of by contact at I.

Hence, *if* the wheels be constructed with teeth formed to these *involute curves,* they will work with perfect regularity of motion.

In practice, the chief condition to be observed is to diminish the pressure on the axes, which is the chief defect of this form of teeth. The common tangent should be drawn through I, making an angle with C_1C_2, not deviating more than 20° from a right angle. Involute wheels have the double advantage that they work equally well if, through the wear of the brasses, the wheels have receded from one another; and any involute wheels of the same pitch and similarly described—that is, having the common tangent to the base circles passing through the point of contact of the pitch lines, or, in other words, base circles proportional to the primitive radii —will work together.

Mr. Hawkins, the translator of *Camus*, first proposed a simple instrument for describing the teeth of wheels to an involute curve. It consists of a straight piece of watch-spring MP (fig 56), with a

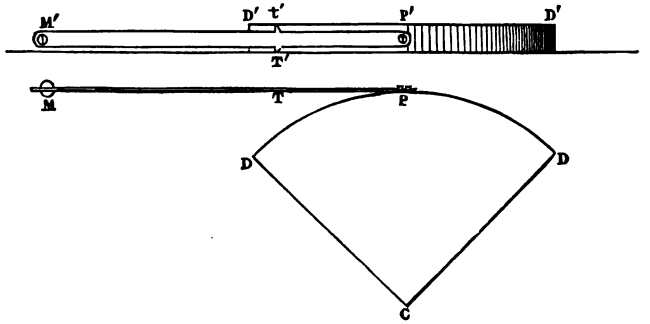


Fig. 56.

screw at one end, and filed away at the edges so as to leave two teeth or tracers, t',T', projecting from the edges of the watch-spring. At M a bit of wire is put through and riveted, so as to form a knot by which the spring can be firmly held and stretched as it is unwound from the base on which the involute is generated. This watch-spring is screwed to the edge of the templet curved to the radius of the base circle of the involute; and this being placed so that its centre coincides with the centre of the wheel, and revolved to bring one of the tracing points t',T', in succession to each of the points at which corresponding faces of the teeth cut the pitch line, a series of involute curves may be described by unfolding the watch-spring, whilst keeping it firmly stretched tangentially to the sector to which it is fixed. The sector must then be turned over, and the involutes of the opposite faces of the teeth struck in a similar manner.

Another *plan is to* employ a straight ruler instead of the watch-

spring, a tracer being fixed in its edge. This shews that the involute is an epicycloid generated by a straight line. The ruler must be kept in contact with the base circle, and the tracer brought in succession to all the points to which the faces of the teeth cut the pitch line.

Hence, to describe a wheel with involute teeth, the line of centres must be drawn and divided proportionally to the number of teeth in the wheel and pinion. Draw the pitch line; divide the pitch line into the same number of equal parts as there are teeth in the wheel, and at these points mark out the thickness of the teeth all round. Draw the tangent to the base circles, making an angle of about 80° with the line of centres, which will give the radius of the base circle drawn touching it. A templet must be made to the radius, and then the involutes may be drawn by either of the preceding methods.

Allowance must be made to permit free play of the teeth in the spaces, the teeth being somewhat shorter than the distance between the bases of the involutes. But wheels of this figure require but little play in the engagement.

In the case of racks, the rack-teeth are bounded by straight lines perpendicular to the tangent, drawn from the point where the pitch lines touch the base circle from which the involutes of the wheel are struck. If the teeth of the rack be made rectangular—that is, bounded by lines perpendicular to the pitch line—the involute must be struck from a base circle equal to the pitch circle of the wheel. In the former case there is a downward pressure on the rack; in the latter, the teeth of the wheel touch those of the rack in a single point—viz., the pitch line of the latter.

Professor Willis's Method of Striking the Teeth of Wheels.

In practice, the custom of describing the teeth of wheel as arcs of circles has from its simplicity been generally adopted. The methods already given, however simple, when adopted in the formation of a single tooth, become tedious in their application to wheels of large size; and to this must be added the imperfect comprehension of their advantages by the millwrights charged with the task of designing wheel patterns.

Circular arcs struck at random, according to the judgment of the millwright, are often employed; and even where better principles have been introduced, it is common, after describing a *single tooth* accurately, to find by trial a circular arc nearly corre-

sponding with its curve, and to employ this in marking out the cogs of the required wheel.

Seeing the advantages of the circular arc, and believing that it is not objectionable, if only the employment of it is guided by true principles, Professor Willis has rendered this great service to practical mechanics—he has shewn how, by a simple construction, the arcs of circles may be found, which, used in the construction of the teeth of wheels, will work truly on each other.

Let AB, fig. 57, be the centres of a wheel and pinion, and C the point of contact of the pitch circles on the line of centres. Through C draw cCc' at any angle with AB. Assume c as the centre from which to describe an arc for a tooth of the wheel A. Draw CD perpendicular to cCc', and from A through c draw AcD, meeting CD in D.

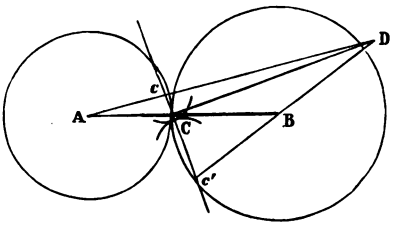


Fig. 57.

Lastly, from D through B draw DBc', meeting cCc' in c'. Then a small arc drawn from c with radius cC as a tooth for the wheel A, will work correctly with a small arc drawn from c', with a radius $c'C$ as a tooth for the wheel B.

Professor Willis recommends 75° 30′ as the best magnitude of the angle ACc, so that $\cos 75° 30' = \frac{1}{4}$. If this angle be constant in a set of wheels, any two will work truly together.

For the easier description of these teeth, Professor Willis has invented the odontograph, a simple instrument of graduated card or wood, by which the position of the centres and radii of the arcs of the teeth can very easily be found, which is of the form shewn in fig. 58, of half its proper lineal dimensions.

THE ODONTOGRAPH.

SHEWING THE CENTRES AND RADII OF THE ARCS OF TEETH.

TABLES SHEWING THE PLACE OF THE CENTRES UPON THE SCALES.

Centres for the flanks of teeth.

No. of teeth.	Pitch in inches.							
	1	1¼	1½	1¾	2	2¼	2½	3
13	129	160	193	225	257	289	321	386
14	69	87	104	121	139	156	173	208
15	49	62	74	86	99	111	123	148
16	40	50	59	69	79	89	99	191
17	34	42	50	59	67	75	84	101
18	30	37	45	52	59	67	74	89
20	25	31	37	43	49	56	62	74
22	22	27	33	39	43	49	54	65
24	20	25	30	35	40	45	49	59
26	18	23	27	32	37	41	46	55
30	17	21	25	29	33	37	41	49
40	15	18	21	25	28	32	35	42
60	13	15	19	22	25	28	31	37
80	12	...	17	20	23	26	29	35
100	11	14	...	...	22	25	28	34
150	...	13	16	19	21	24	27	32
Rack	10	12	15	17	20	22	25	30

Centres for the faces of teeth.

12	5	6	7	9	10	11	12	15
15	...	7	8	10	11	12	14	17
20	6	8	9	11	12	14	15	18
30	7	9	10	12	14	16	18	21
40	8	...	11	13	15	17	19	23
60	...	10	12	14	16	18	20	25
80	9	11	13	15	17	19	21	26
100	...	...	...	...	18	20	22	...
150	...	...	14	16	19	21	23	27
Rack	10	12	15	17	20	22	25	30

The figure is of half the linear dimensions of the original, *which may be obtained of Messrs. Holtzapfel, of London.*

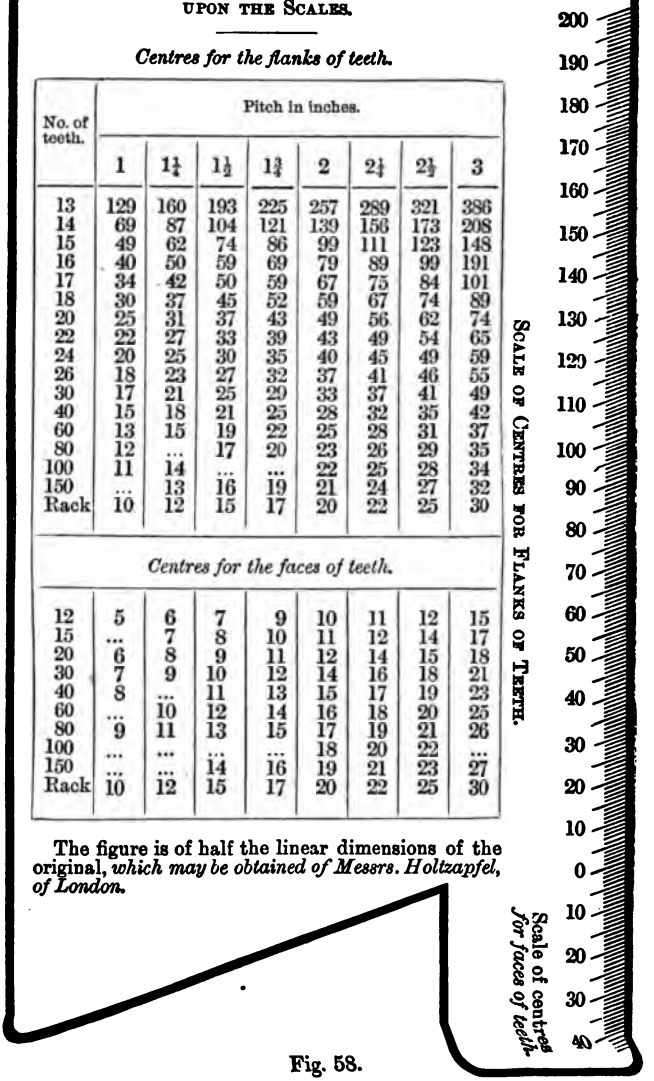


Fig. 58.

It has the bottom edge bevelled off at an angle of 75°. The point where this would cut the right-hand edge is the zero of the scales. These scales are graduated to twentieths of an inch, to avoid fractional parts in the tables, and depart in each direction from the zero, the upper being that employed in finding the centres of the flanks of the teeth or parts within the pitch circle, and the lower for finding the centres of the faces of the teeth or parts without the pitch circle. Tables are given on the odontograph for finding the graduation on the scale corresponding to any given pitch and number of teeth. For intermediate pitches not given in the table, or for wheels of greater size, the corresponding numbers can be found by simple proportion. For wheels of only twelve teeth the flanks are straight, and form parts of radii of the pitch circle.

In the accompanying figure let A be the centre of a wheel, K*d*L the pitch line. Set off KL equal to the pitch, and bisect it in *d*. Draw radii AK, AL. Place the odontograph with its bevelled edge on the radius AK, and zero of the scale on the pitch line. Then look out in the table of centres for the flanks of teeth, the number corresponding to the pitch, and required number of teeth, and mark off this point *h* from the scale of centres for the flanks of teeth. Then remove the odontograph, and similarly place it on the radius AL. Find in the table of centres for the faces of teeth in the wheel, and mark it off at *f* on the scale for centres of the faces of teeth. Then describe two arcs from *h* and *f*, with *hd* and *fd* as radii; these will form the side of the tooth. Then, from *d* let the pitch line be marked off into as many equal parts as there are teeth in the wheel, and those be divided proportionally to the widths of the teeth and spaces. Through *h* and *f*, with radii A*h* and A*f*, draw circles. Take *hd* as a radius, and placing one foot of the compass on the divisions of the pitch line and the other in the circle drawn through, describe a series of arcs forming the flanks of the teeth. Similarly with radius *fd*, and one leg of the compass on the circle drawn through *f*, describe the faces of the teeth.

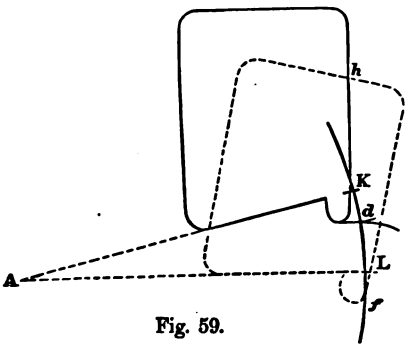


Fig. 59.

For an annular wheel the same rules apply, only that the part *of the curve which is face* in a spur wheel becomes the flank in an

annular wheel, and *vice versâ*. For a rack the pitch line is straight, and AK, AL are parallel and perpendicular to it, at a distance equal to the pitch. As these odontographs may be purchased in a very convenient form, with tables for their use, and also with tables of the widths of teeth, and spaces and length of teeth within and without the pitch circle, it is not necessary to describe them in further detail.

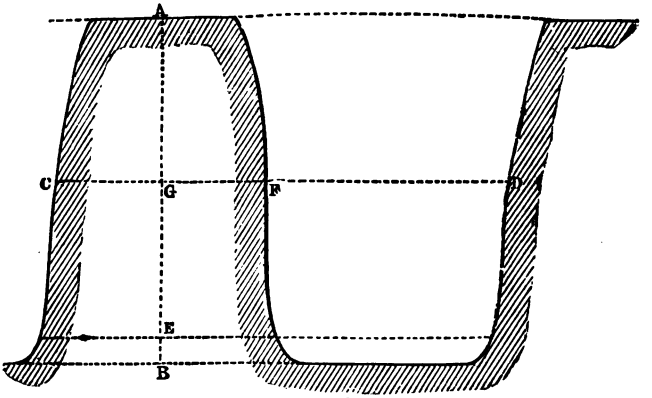


Fig. 60.

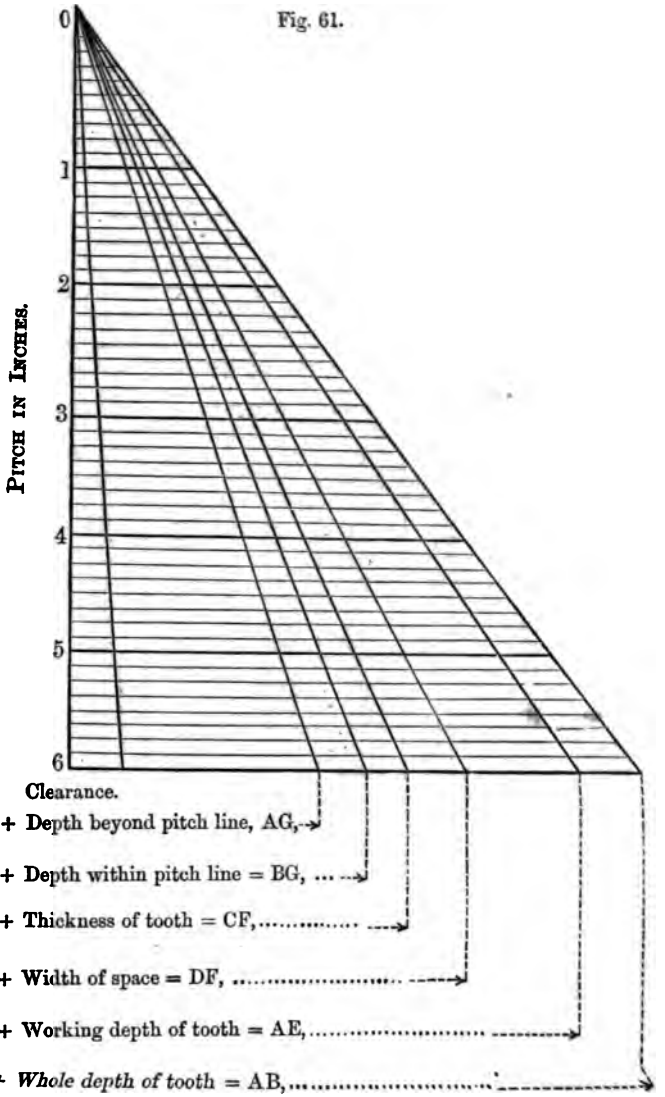
The proportions of the parts of teeth of wheels may be given as follows :—

			Proportional part.		Inches.
Pitch, ...	= CD	=	1·00	=	$2\frac{1}{2}$
Depth, ...	= AB	=	0·75	=	$1\frac{7}{8}$
Working depth, . . .	= AE	=	0·70	=	$1\frac{3}{4}$
Clearance, ...	= EB	=	0·05	=	$\frac{1}{8}$
Thickness, ...	= CF	=	0·45	=	$1\frac{1}{8}$
Width of space, ...	= FD	=	0·55	=	$1\frac{3}{8}$
Play, or FD – CF, . . .	=	=	0·10	=	$\frac{1}{4}$
Length beyond pitch line,	= AG	=	0·35	=	$\frac{7}{8}$

Taking these proportions, we may construct a scale which shall give directly the corresponding numbers for any pitch. Taking a vertical line, and dividing it into eighths of an inch, we get the scale of pitches (fig. 61). Draw lines perpendicular to this, and on any one of them mark off a series of distances equal to the clearance, depth, thickness, &c., of the teeth corresponding to that pitch. Through *o* and these points draw the lines shewn in the figure; they will divide the lines corresponding to all other pitches *in the same proportion*.

SCALE OF PROPORTIONS.

Fig. 61.



It is usual to allow a greater amount of clearance in small wheels than is necessary in large ones. Very varying proportions have been given by different millwrights—$\frac{1}{10}$, $\frac{1}{12}$, $\frac{1}{15}$, and $\frac{1}{20}$ of the pitch having been used in different circumstances, even with the best millwork. In the scale (fig. 61) this has to a certain extent been taken into account—$\frac{1}{10}$ of the pitch is allowed in smaller wheels, decreasing to $\frac{1}{15}$ in the largest; hence the lines are not absolutely straight, but are slightly curved, except that for the whole depth of the tooth, which quantity has been assumed to vary directly as the pitch.

Assuming that this scale represents with sufficient accuracy the proportions which practice shews to be best in average cases, we may construct a table for the guidance of the millwright. From this he must vary in cases where it appears necessary to allow more for defects of workmanship, or to permit less "backlash" or reaction, it being understood that the table will only apply in cases where the teeth are formed with an approximation to the true mathematical figure.

TABLE OF PROPORTIONS OF TEETH OF WHEELS FOR AVERAGE PRACTICE.

Pitch.	Clearance and play.	Depth beyond pitch line.	Depth within pitch line.	Working depth.	Whole depth.	Thickness of tooth.	Width of space.
$\frac{1}{2}$	·06	·16	·22	·32	·38	·22	·28
$\frac{3}{4}$	·08	·25	·33	·50	·58	·33	·42
1	·10	·335	·435	·67	·77	·45	·55
$1\frac{1}{4}$	·12	·42	·54	·84	·96	·56	·69
$1\frac{1}{2}$	·13	·51	·64	1·02	1·15	·68	·82
$1\frac{3}{4}$	·14	·60	·74	1·20	1·34	·80	·95
2	·16	·685	·845	1·37	1·53	·92	1·08
$2\frac{1}{4}$	·17	·775	·945	1·55	1·72	1·04	1·21
$2\frac{1}{2}$	·19	·86	1·05	1·72	1·91	1·15	1·35
$2\frac{3}{4}$	·20	·95	1·15	1·90	2·10	1·27	1·47
3	·22	1·04	1·26	2·08	2·30	1·39	1·61
$3\frac{1}{4}$	·23	1·13	1·36	2·26	2·49	1·51	1·74
$3\frac{1}{2}$	·25	1·215	1·465	2·43	2·68	1·62	1·88
$3\frac{3}{4}$	·26	1·305	1·565	2·61	2·87	1·74	2·01
4	·28	1·39	1·67	2·78	3·06	1·86	2·14
$4\frac{1}{2}$	·31	1·565	1·875	3·13	3·44	2·09	2·40
5	·34	1·745	2·085	3·49	3·83	2·33	2·67
$5\frac{1}{2}$	·37	1·925	2·295	3·85	4·21	2·56	2·93
6	·40	2·10	2·50	4·20	4·60	2·80	3·20

In wood and iron gear where the teeth are carefully cut, very little, if any, clearance is necessary, as they work much better when the tooth of each wheel fills its allotted space. It is, however, different where wheels have to gear together direct from the *foundry, where* the teeth are not unfrequently deranged in the act *of moulding in* the sand.

This table (the preceding one) gives the number to the nearest hundredth of an inch. It may be converted into the ordinary scale of eighths by the following table:—

	Thirty seconds of an inch.										Remarks.
	1	2	3	4	5	6	7	8	9	10	See preceding Table.
Corresponding Decimal.	·031	·062	0·94	·125	·156	·188	·219	·250	·281	·3125	

As, unfortunately, decimal scales are not yet much used by millwrights, the following table has been prepared, giving the numbers in the preceding table in the thirty-seconds of an inch, such changes being made as will reduce as much as possible the errors of employing this *rough* standard. The former table is to be preferred where it can be relied on. The left-hand figures in each column are inches, the right-hand ones thirty-seconds of an inch, the denominators of the fraction being omitted.

TABLE GIVING THE PROPORTIONS OF TEETH OF WHEELS IN INCHES AND THIRTY SECONDS OF AN INCH.

Pitch inches	Clearance	Depth beyond the pitch line	Depth within the pitch line.	Working depth.	Whole depth.	Thickness of tooth.	Remarks.
½	0″2	0″5	0″7	0″10	0″12	0″7	
⅝	0″3	0″8	0″11	0″16	0″19	0″10	
1	0″3	0″11	0″14	0″22	0″25	0″14	
1¼	0″4	0″13	0″17	0″26	0″30	0″18	
1½	0″4	0″16	0″20	1″0	1″4	0″21	
1¾	0″4	0″19	0″23	1″6	1″10	0″25	
2	0″5	0″22	0″27	1″12	1″17	0″29	
2¼	0″5	0″25	0″30	1″18	1″23	1″1	
2½	0″5	0″28	0″33	1″24	1″29	1″5	See preceding Table.
2¾	0″6	0″31	0″37	1″30	2″4	1″8	
3	0″7	1″1	1″8	2″2	2″9	1″12	
3¼	0″7	1″4	1″11	2″8	2″15	1″16	
3½	0″8	1″7	1″15	2″14	2″22	1″20	
3¾	0″8	1″10	1″18	2″20	2″28	1″23	
4	0″9	1″12	1″21	2″24	3″1	1″27	
4½	0″10	1″18	1″28	3″4	3″14	2″3	
5	0″11	1″24	1″35	3″16	3″27	2″10	
5½	0″11	1″30	1″41	3″28	4″7	2″18	
6	0″12	2″4	2″16	4″8	4″20	2″25	

BEVEL WHEELS.

Hitherto we have considered only that case of toothed wheels in which the pitch lines are in one plane. We have now to examine the modifications which are necessary when the axes of the wheel and pinion are inclined. If, therefore, teeth be applied to these

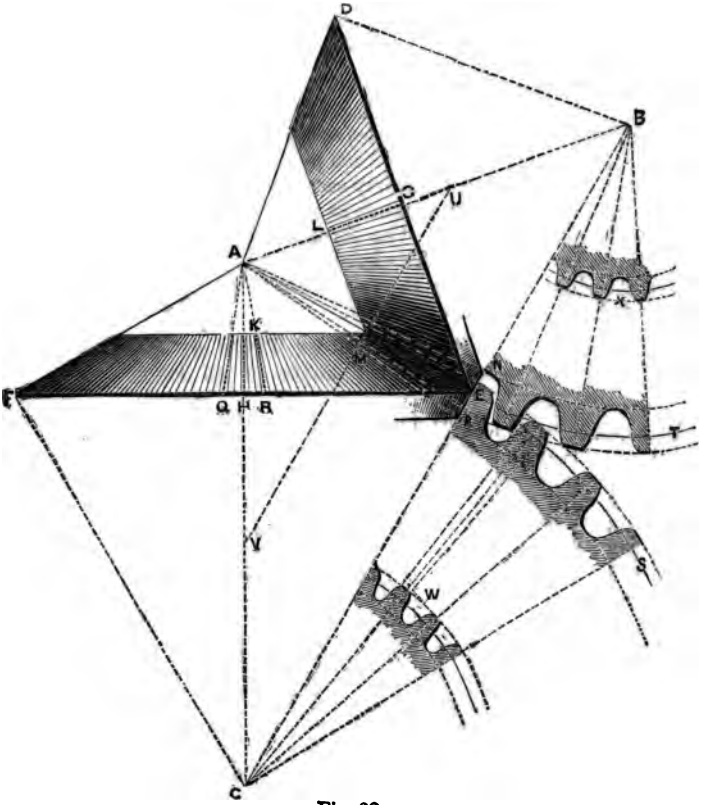


Fig. 62.

frusta in the same manner as in spur gearing they are attached to cylindrical surfaces, bevel gearing will be formed acting on the same principles of sliding contact.

Imagine the surface of two frusta of cones to be unwrapped so *as to lie in one plane*, they will form parts of circular annuli. *Thus, let AFE, ADE* (fig. 62) be two conical frusta; draw BEC

perpendicular to the line of contact AE. From *c*, with the radii as shewn, describe the three circles; and from B, with the radii as shewn, describe the other similar circles; then the surfaces EPS and ENT will be developments of the frusta EF, ED. Let these be treated as spur wheels, and ES, ET being treated as the pitch lines, let teeth be described by a describing circle in the method already explained for epicycloidal or other teeth. If, then, the plane on which these have been described, and which we suppose of drawing paper or other flexible material, be cut along the outer and inner arcs, the circular annuli may be wrapped round the frusta EF, ED, and the forms of the teeth traced off upon them.

The axes of bevel wheels are in practice, in the great generality of cases, at right angles.

THE WORM AND WHEEL.

By this contrivance the motion of a screw is communicated with great smoothness to oblique teeth on a spur wheel.

The section of a screw through its axis is precisely similar to that of a double rack. Let AB be such a section, and for simplicity suppose that the form of the threads of the screw has been determined by one of the rules already given for racks. Then the teeth of the wheel CDE may evidently be formed so as to work with the centre section of the screw. Now the effect of the revolu-

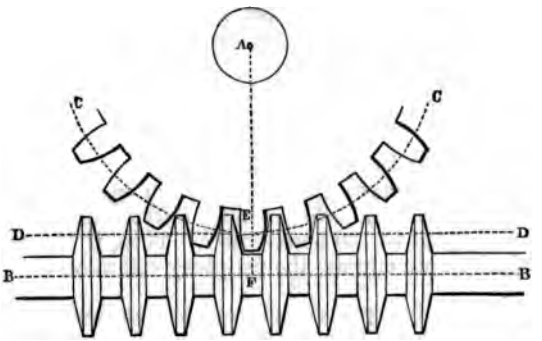


Fig. 63.

tion of the screw is precisely similar to that of the racks, and the sections of the threads of the screw will appear to travel from end to end, in the same way as a rack pushed forward in the same *direction.*

If, therefore, it is sufficient that the wheel teeth be in contact with the screw at one point only, the teeth of the wheel may be made oblique, but straight, the obliquity being equal to the pitch of the screw. This is the usual practice of millwrights. If, however, the teeth are required to be in contact with the entire breadth of the tooth, the outline of the tooth must vary in every section of the wheel, and the process of describing these teeth becomes very complex. Practically, the difficulty has been overcome, by first making a pattern screw of steel, notched in the threads to convert it into a cutting instrument. The wheel is then roughly cut out, and being fixed in a frame, the screw is used to cut out the spaces between the teeth to their true form.

STRENGTH OF THE TEETH OF WHEELS.

The pressure on the teeth varies directly as the horse-power transmitted, and inversely as the velocity of revolution. Thus, if one wheel transmit 5 horse-power and another 10 horse-power at the same velocity, the strain on the latter will be twice that on the former. Or, again, if two wheels transmit the same power, but one at a velocity of 100 feet per minute, and the other at only 25 feet per minute, the strain on the former will be only one fourth that on the latter.

Let V be the velocity in feet per second, H the number of horse-power transmitted, then the total pressure on the wheels will be—

$$P = \frac{550H}{V},$$

where P is the statical pressure in lbs.

For example, suppose the fly-wheel of an engine to be 24 feet in diameter, and to work into a pinion 5 feet diameter, and let the work transmitted be 150 horse-power. Then, if the wheel makes 25 revolutions per minute, the periphery will move at a velocity of $\frac{75{\cdot}4 \times 25}{60} = 31{\cdot}4$ feet per second; and the statical pressure on the teeth will be $\frac{550 \times 150}{31{\cdot}4} = 2627$ lbs.

In addition to statical pressure, however, a different element has *to be taken into* account—namely, the impacts due to sudden

accelerations or retardations of speed. The allowance which must be made to prevent accident from this cause varies exceedingly in different kinds of machinery. It is great in the gearing of rolling mills, for instance, and in all machinery in which the strains are irregular.

In calculating the strength of the tooth, it has been usual to consider it as a short beam fixed at one end, and having the whole of the pressure applied along the extremity of the tooth. But there is a position in which the teeth may be subjected to a severer stress still; owing to the wear of brasses and teeth, we cannot calculate upon the strain bearing always on the whole breadth of the tooth. The pressure may not only come on the extremity of a tooth, but if any obstruction come in between the teeth, it may be thrown entirely upon one corner of the tooth. In such a case it may be shewn, by the rules of maxima and minima, that if EC = CB, the greatest stress will be near the line EB.

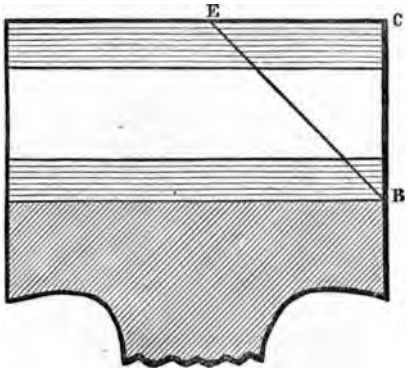


Fig. 64.

Mr. Tredgold has expressed the strength of a tooth on this supposition by the formula—

$$W = \frac{fd^2}{5},$$

where d is the thickness of the tooth. To allow for wear, however, he adds one third, so that

$$W = \frac{fd^2\,(1 - \frac{1}{3})^2}{5} = \frac{fd^2}{10{\cdot}25}$$

In cast iron $f = 15{,}300$, and hence

$$d = \sqrt[2]{\frac{W}{1500}}$$

Or in words, the thickness necessary for the tooth or inches is equal to the square root of the stress on the tooth in pounds divided by 1500. Hence Tredgold has computed the following table, the breadths of the teeth being deduced, on the principle that the stress should not exceed 400 lbs. per inch breadth :—

TABLE OF THICKNESS, BREADTH, AND PITCH OF WHEELS OF TEETH.

Stress in lbs. at the Pitch Line.	Thickness of Teeth in Inches.	Breadth of Teeth in Inches.	Pitch in Inches.
400	0·52	1	1·1
800	0·73	2	1·5
1200	0·90	3	1·9
1600	1·03	4	2·2
2000	1·15	5	2·4
2400	1·26	6	2·7
2800	1·36	7	2·9
3200	1·46	8	3·0
3600	1·56	9	3·3
4000	1·64	10	3·4
4400	1·70	11	3·6
4800	1·78	12	3·7
5200	1·86	13	3·9
5600	1·93	14	4·0
6000	2·00	15	4·2

To use this table, when the horses' power transmitted by the wheel are known, the reader must refer to the table on page 129.

Elsewhere, Mr. Tredgold has given a rule of the following description :—

$$d = \tfrac{3}{4}\sqrt{\frac{H}{v}} \text{ for cast iron,}$$

where d is the requisite thickness of a tooth to transmit a force of (H) horses at a velocity (v) feet per second.

Hence, Tredgold's last rule for the thickness of cast-iron teeth is as follows:—"Find the number of horses' power transmitted by *the wheel, and* divide that number by the velocity in feet per *second of the pitch* line of the pinion or wheel; extract the square

root of the quotient, and three-fourths of this root will be the least thickness of cast-iron teeth for the wheel or pinion." From this he derives a second rule for the pitch, which manifestly depends on the thickness of the tooth, namely, multiply the thickness of the tooth by 2·1, and the product will be the pitch. The same result may be obtained from inspection of the tables I have given at page 120. Wooden teeth he recommends to be made of twice the thickness of cast-iron ones. But one-and-a-half times the thickness is a sufficient allowance.

A writer in the *Engineer and Machinists' Assistant* deduces another, but equally simple rule, for the thickness of teeth; he assumes the relation—

$$t = c\sqrt{w},$$

where t is the thickness of the tooth, w the pressure on the tooth, and c a constant depending on the nature of the material. Let, then, a be the strength of a bar 1 inch long, 1 broad, and 1 thick. Then, to support a weight (w) by a bar of a length (l), and breadth (b)—

$$t = \sqrt{\frac{w \times l}{a \times b}}$$

Suppose the breadth of the tooth to be fixed at twice its length—

$$t = \sqrt{\frac{wl}{a \times 2l}} = \sqrt{\frac{w}{2a}}$$

Taking $a = 8000$ lbs. for cast iron, $2a = 16{,}000$ lbs., but as this is the breaking weight the safe working-pressure will be only 1600 lbs., and the thickness of the tooth for safe working will be, for cast iron,—

$$t = \sqrt{\frac{w}{1600}} = 0{\cdot}025\sqrt{w},$$

where w being given in pounds t is found in inches. Similarly for other materials he obtains,—

$c = {\cdot}035$ for brass,
$= {\cdot}038$ for hardwood.

For example, in the wheel assumed at page 124, w was found to

be 2627 lbs. Hence the necessary thickness of the tooth, if of cast iron, would be $\cdot 025 \sqrt{2627} = 1 \cdot 28$ inches. Referring to the tables of the relation of pitch, &c., the teeth of 2·1 inches length, and the breadth of the wheel $2 \cdot 1 \times 2 = 4 \cdot 2$ inches at the least. By Tredgold's latter rule, the thickness of the teeth for the same wheel would be—

$$t = \tfrac{3}{4} \sqrt{\frac{150}{31 \cdot 4}} = 1 \cdot 41 \text{ inches};$$

$$\text{the pitch, } = 2 \cdot 1 \times 1 \cdot 41 = 3 \cdot 0 \text{ inches};$$

$$\text{and the breadth, } = \frac{2627}{400} = 6\tfrac{1}{2} \text{ inches};$$

bearing in mind that $w = \dfrac{550\text{H}}{v}$, where H is the maximum horse-power transmitted, and v the velocity of the pitch-line of the wheel in feet per second, we may give these formulæ in a more convenient form—

$$t = x \sqrt{\frac{\text{H}}{v}}$$

Where $x = 0 \cdot 587$ for cast iron,
„ $x = 0 \cdot 821$ for brass,
„ $x = 0 \cdot 891$ for wood,

conversely, if a wheel having teeth t inches thick be given, the horse-power it is capable of transmitting is given by the formula—

$$\text{H} = t^2 \frac{v}{x^2}$$

Where $x^2 = 0 \cdot 344$ for cast iron,
„ $= 0 \cdot 674$ for brass,
„ $= 0 \cdot 795$ for wood.

From the following table the pressure with different velocities, and amount of horse-power, may be obtained by interpolation, remembering that the pressure varies inversely as the former, and directly as the latter. To this we have appended another table, giving the horses' power which can be safely transmitted by wheels of different pitches, when proportioned according to the table at page 126. The last of these tables has been calculated on *the assumption* that 400 lbs. per inch is the greatest working stress *which is consistent* with durability in ordinary cases.

Relation of Horses'-power transmitted, and Velocity at the Pitch Circle to pressure on Teeth.

Number of horses' power transmitted.	Velocity in feet per second.									
	1 ft.	3 ft.	5 ft.	7 ft.	9 ft.	11 ft.	13 ft.	15 ft.	20 ft.	25 ft.
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1	550	183	110	79	61	56	42	37	28	22
2	1100	367	220	157	122	100	85	73	55	44
3	1650	550	330	236	183	150	127	110	83	66
4	2200	733	440	314	244	200	169	146	110	88
5	2750	917	550	393	306	250	212	183	138	110
10	5500	1833	1000	786	611	500	423	367	275	220
15	8250	2750	1650	1179	927	750	635	550	413	330
20	11000	3666	2200	1571	1222	1000	846	733	550	440
25	13750	4583	2750	1964	1527	1250	1058	917	688	550
30	16500	5500	3300	2357	1833	1500	1269	1000	825	660
40	22000	7333	4400	3143	2444	2000	1692	1467	1100	880
50	27500	9167	5500	3928	3055	2500	2115	1833	1375	1100
60	33000	11000	6600	4714	3667	3000	2538	2200	1650	1320
70	38000	12833	7700	5500	4278	3500	2962	2567	1925	1540
80	44000	14667	8800	6285	4889	4000	3385	2933	2200	1760
90	49500	16500	9900	7071	5500	4500	3808	3308	2475	1980
110	60500	20167	12100	8643	6722	5500	4654	4033	3025	2420
120	66000	22000	13200	9423	7333	6000	5077	4400	3300	2640
130	71500	23833	14300	10214	7944	6500	5500	4767	3575	2860
140	...	25667	15400	11000	8556	7000	5923	5133	3850	3080
150	...	27500	16500	11786	9167	7500	6346	5500	4125	3300
160	...	29333	17600	12571	9778	8000	6769	5867	4400	3520
170	...	31167	18700	13357	10389	8500	7192	6233	4675	3740
180	...	...	19800	14143	11000	9000	7615	6600	4950	3960
190	...	...	20900	14929	11611	9500	8038	6967	5225	4180
200	...	...	22000	15714	12222	10000	8462	7333	5500	4400
300	...	...	33000	23571	18333	15000	12692	7700	8250	6600
400	...	...	44000	31428	24444	20000	16923	8067	11000	8800
500	...	...	55000	39285	30555	25000	21154	8433	13750	11000

TABLE SHEWING THE PITCH AND THICKNESS OF TEETH TO TRANSMIT A GIVEN NUMBER OF HORSES' POWER AT DIFFERENT VELOCITIES.

Pitch in Inches.	Thickness of Teeth in Inches.	Velocity at Pitch Line in feet per second.												
		1	3	5	7	9	11	13	15	20	25	30	40	50
		H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.
⅜	·22	·14	·42	·71	·99	1·26	1·5	1·8	2·1	2·8	3·5	4·2	5·6	7·1
½	·33	·32	·95	1·59	2·22	2·75	3·5	4·1	4·7	6·3	7·9	9·5	12·7	15·9
1	·45	·59	1·77	2·95	4·12	5·31	6·5	7·7	8·8	11·8	14·7	17·7	23·6	29·5
1¼	·56	·91	2·74	4·56	6·38	8·22	10·0	11·9	13·7	18·2	22·8	27·4	36·5	45·6
1½	·68	1·34	4·03	6·72	9·41	12·09	14·8	17·5	20·2	26·9	33·6	40·3	53·7	67·2
1¾	·80	1·86	5·58	9·31	13·03	16·74	20·5	24·2	27·9	37·2	46·5	55·8	74·4	93·1
2	·92	2·46	7·38	12·31	17·23	22·14	27·1	32·0	36·9	49·2	61·5	73·8	98·4	123·1
2¼	1·04	3·14	9·43	15·72	22·01	27·29	34·6	40·9	47·2	62·9	78·6	94·3	125·7	157·2
2½	1·15	3·85	11·54	19·23	26·92	34·62	42·3	50·0	57·7	76·9	96·1	115·4	153·8	192·3
2¾	1·27	4·69	14·07	23·45	32·82	42·21	51·6	61·0	70·3	93·8	117·2	140·6	187·5	234·5
3	1·39	5·62	16·84	28·08	39·31	50·42	61·8	73·0	84·2	112·3	140·4	168·5	224·6	280·8
3¼	1·51	6·63	19·88	33·14	46·40	59·64	72·9	86·2	99·4	132·6	165·7	198·8	265·1	331·4
3½	1·62	7·63	22·89	38·15	53·40	68·67	83·9	99·2	114·4	152·6	190·7	228·9	305·1	381·5
3¾	1·74	8·80	26·40	44·00	61·61	79·20	96·8	114·4	132·0	176·0	220·0	264·0	352·0	440·0
4	1·86	10·06	30·17	50·29	70·40	90·50	110·6	130·7	150·8	201·1	251·4	301·7	402·3	502·9
4½	2·09	12·70	38·09	63·49	88·89	114·27	139·7	165·1	190·5	254·0	317·5	380·9	507·9	634·9
5	2·33	15·78	47·35	78·91	110·47	142·05	173·6	205·2	236·7	315·6	394·5	473·5	631·3	789·1
5½	2·56	19·05	57·15	95·26	133·36	171·45	209·6	267·7	285·8	381·0	476·3	571·5	762·0	952·6
6	2·80	22·79	68·37	113·96	159·54	205·11	250·7	316·3	341·9	455·8	569·7	683·7	911·6	1139·6

TABLE SHEWING THE BREADTH OF TEETH REQUIRED TO TRANSMIT DIFFERENT AMOUNTS OF FORCE AT A UNIFORM PRESSURE OF 400 lbs. PER INCH.

No. of Horses' Power transmitted.	Velocity of Pitch Line in feet per second.										
	1	3	5	7	9	11	13	15	20	25	30
	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
1	1·4	0·5	0·3	0·2	0·15	0·12	0·11	0·09	0·07	0·05	0·05
2	2·7	0·9	0·6	0·4	0·3	0·25	0·21	0·18	0·14	0·11	0·09
3	4·1	1·4	0·8	0·6	0.5	0·38	0·32	0·27	0·21	0·17	0·14
4	5·5	1·8	1·1	0·8	0·6	0·50	0·42	0·37	0·28	0·22	0·18
5	6·9	2·3	1·4	1·0	0·8	0·62	0·53	0·46	0·35	0·28	0·23
10	13·7	4·6	2·7	2·0	1·5	1·3	1·06	0·92	0·69	0·55	0·46
15	20·6	6·9	4·1	2·9	2·3	1·9	1·6	1·37	1·03	0·83	0·69
20	34·2	9·2	5·5	3·9	3·1	2·5	2·1	1·83	1·38	1·10	0·92
25	41·2	11·4	6·9	4·9	3·8	3·1	2·6	2·3	1·72	1·38	1·14
30	55·0	13·8	8·2	5·9	4·6	3·8	3·2	2·8	2·06	1·65	1·38
40	68·8	18·3	11·0	7·9	6·1	5·0	4·2	3·7	2·75	2·20	1·83
50	82·5	22·9	13·7	9·8	7·6	6·3	5·3	4·6	3·44	2·75	2·29
60	96·2	27·5	16·5	11·8	9·1	7·5	6·4	5·5	4·1	3·30	2·75
70	110·2	32·1	19·2	13·8	10·7	8·8	7·4	6·4	4·8	3·85	3·21
80	123·8	36·7	22·0	15·7	12·2	10·0	8·5	7·3	5·5	4·40	3·67
90	137·5	41·2	24·8	17·7	13·8	11·2	9·5	8·2	6·2	4·95	4·12
100	151·2	45·8	27·5	19·6	15·3	12·5	10·6	9·1	6·9	5·50	4·58
110	165·0	50·4	30·3	21·6	16·8	13·7	11·6	10·1	7·5	6·05	5·04
120	178·8	55·0	33·0	23·5	18·3	15·0	12·7	11·0	8·3	6·6	5·50
130	...	59·8	35·7	25·5	19·8	16·3	13·8	11·9	8·9	7·2	5·98
140	...	64·2	38·5	27·5	21·4	17·5	14·8	12·8	9·6	7·7	6·42
150	...	68·7	41·3	29·4	22·9	18·7	15·8	13·7	10·3	8·2	6·87
160	...	73·3	44·0	31·4	24·5	20·0	16·9	14·6	11·0	8·8	7·33

PART V.—WHEEL AND SCREW CUTTING.

WHEEL CUTTING

Definitions.

A WHEEL-CUTTING machine is used to cut teeth in wheels of different pitches and forms, as may be required; and where great accuracy is required in a machine, the wheels are generally cut instead of cast.

A wheel-cutting machine has a wheel keyed on the opposite end of the mandril to that where the wheel is fixed while undergoing the operation of being cut, and is called the *Mandril Wheel.* This wheel has 180 and sometimes 240 teeth, and is geared into and driven by a worm on a spindle forming a tangent with the mandril wheel; a *Change Wheel* is attached to one end of this spindle (commonly called the *Tangent Wheel*), but which may more properly be known as the *Driven Wheel.* An intermediate wheel gears this together, known as the *Division Plate Wheel,* which should be called the *Driver,* for this intermediate wheel only answers the purpose of gearing them together, and to simply reverse the motion of the tangent spindle. One tooth being cut, the motion is given to the *Driver* with a handle and lever, which drops into a notch, when one, two, or three turns have been given, as the case may require.

Rules.—The Change Wheels generally run up in single numbers *from 21 to 80 teeth,* and the general rule of working, when

practicable, is to place *a wheel having the same number of teeth as the one required to be cut* upon the tangent spindle-end (to be the driven wheel), and the 80 as the driving wheel when the mandril wheel has 240 teeth in, as per example, $80 \times 3 = 240$, *with three turns of the handle or lever.* When the mandril wheel has 180 teeth in, act precisely in the same manner as above, only instead use 60 as the driving wheel, as, for example, $60 \times 3 = 180$.

Examples.

(1.) Find the change wheels to cut a wheel which requires 55 teeth in, with a mandril wheel having 240.

Here we have 80 as the "driver" and 55 as the "driven" wheels, with three turns of the handle, which gives the *55th part of* 240 every three turns of the driver, or one tooth in the wheel to be cut.

(2.) Find the change wheels to cut a wheel which requires 55 teeth in, with a mandril wheel having 180 teeth.

Here we have 60 as the driver and 55 as the driven wheels, with three turns of the handle, same as in foregoing example, and working with any intermediate wheel that will gear suitably.

When there is not a complete set of change wheels, and the before-mentioned rules are not practicable, place the number of teeth in the mandril wheel and wheel to be cut in a fractional form, place the number of teeth in the mandril wheel as the numerator, and the number of teeth required to be cut as the denominator, then divide both numerator and denominator by 2, 4, 6, 8, 9, &c., or any other number convenient, as long as the proportion is kept in the same ratio.

Example.—Find the change wheels to cut a wheel which requires 100 teeth, the mandril wheel having 240 teeth.

$$\frac{4)240}{100}\begin{cases}60 \text{ "Driver,"} \\ 25 \text{ "Driven,"}\end{cases}$$

with one turn of the handle or "driving wheel," when this rule *is used.*

TABLE OF CHANGE WHEELS TO BE USED AT A WHEEL-CUTTING MACHINE, THE MANDRIL WHEEL HAVING 240 TEETH.

No. of teeth.	Driver.	Driven.	No. of turns.	No. of teeth.	Driver.	Driven.	No. of turns.	No. of teeth.	Driver.	Driven.	No. of turns.	No. of teeth.	Driver.	Driven.	No. of turns.	No. of teeth.	Driver.	Driven.	No. of turns.
10	..	..	..	72	60	36	2	134	60	67	2	196	60	49	..	258	40	43	1
11	80	24	6	73	80	73	3	135	80	45	1	197	..	..	..	259	..	..	..
12	80	26	6	74	60	37	2	136	60	34	1	198	40	33	1	260	48	52	1
13	80	28	6	75	80	25	1	137	..	..	..	199	..	..	..	261	..	..	..
14	80	30	6	76	60	38	2	138	80	46	1	200	60	50	1	262	..	..	..
15	80	32	6	77	80	77	3	139	..	..	..	201	80	67	1	263	..	..	..
16	80	34	6	78	78	60	3	140	60	35	1	202	..	..	..	264	30	44	1
17	80	36	6	79	80	79	3	141	80	47	1	203	..	..	..	265	48	53	1
18	80	38	6	80	60	40	2	142	60	71	2	204	60	51	1	266	..	..	..
19	80	40	6	81	80	27	1	143	..	..	..	205	48	41	1	267	..	..	..
20	80	21	3	82	60	41	2	144	80	48	1	206	..	..	..	268	60	67	1
21	80	22	3	83	..	..	..	145	..	..	..	207	..	..	..	269	..	..	..
22	80	23	3	84	60	42	2	146	60	73	2	208	60	52	1	270	48	54	1
23	80	24	3	85	..	..	..	147	80	49	1	209	..	..	..	271	..	..	..
24	80	25	3	86	60	43	2	148	60	37	1	210	48	42	1	272	60	68	1
25	80	26	3	87	80	29	1	149	..	..	..	211	..	..	..	273	..	..	..
26	80	27	3	88	60	44	2	150	40	25	1	212	60	53	1	274	..	..	..
27	80	28	3	89	..	..	..	151	..	..	..	213	80	71	1	275	48	55	1
28	80	29	3	90	60	45	2	152	60	38	1	214	..	..	..	276	60	69	1
29	80	30	3	91	..	..	..	153	80	51	1	215	48	43	1	277	..	..	..
30	80	31	3	92	60	46	2	154	60	77	2	216	60	54	1	278	..	..	..
31	80	32	3	93	80	31	1	155	..	..	..	217	..	..	..	279	..	..	..
32	80	33	3	94	60	47	2	156	60	39	1	218	..	..	..	280	48	56	1
33	80	34	3	95	..	..	..	157	..	..	..	219	80	73	1	281	..	..	..
34	80	35	3	96	60	48	2	158	60	79	2	220	60	55	1	282	40	46	1
35	80	36	3	97	..	..	..	159	80	53	1	221	..	..	..	283	..	..	..
36	80	37	3	98	60	49	2	160	60	40	1	222	80	74	1	284	60	71	1
37	80	38	3	99	80	33	1	161	..	..	..	223	..	..	..	285	48	57	1
38	80	39	3	100	60	25	1	162	40	27	1	224	60	56	1	286	..	..	..
39	80	40	3	101	..	..	..	163	..	..	..	225	80	75	1	287	..	..	..
40	80	41	3	102	60	51	2	164	60	41	1	226	..	..	..	288	60	72	1
41	80	42	3	103	..	..	..	165	80	55	1	227	..	..	..	289	..	..	..
42	80	43	3	104	60	26	1	166	..	..	..	228	60	57	1	290	48	58	1
43	80	44	3	105	80	35	1	167	..	..	..	229	..	..	..	291	..	..	..
44	80	45	3	106	60	53	2	168	60	42	1	230	48	46	1	292	60	73	1
45	80	46	3	107	..	..	..	169	..	..	..	231	80	77	1	293	..	..	..
46	80	47	3	108	60	27	1	170	48	34	1	232	60	58	1	294	40	49	1
47	80	48	3	109	..	..	..	171	80	57	1	233	..	..	..	295	48	59	1
48	80	49	3	110	60	55	2	172	60	43	1	234	80	78	1	296	60	74	1
49	80	50	3	111	80	37	1	173	..	..	..	235	48	47	1	297	..	..	..
50	80	51	3	112	60	28	1	174	80	58	1	236	60	59	1	298	..	..	..
51	80	52	3	113	..	..	..	175	48	35	1	237	80	79	1	299	..	..	..
52	80	53	3	114	80	38	1	176	80	44	1	238	..	..	..	300	48	60	1
53	80	54	3	115	48	23	1	177	80	59	1	239	..	..	..	301	..	..	..
54	80	55	3	116	60	29	1	178	..	..	..	240	60	60	1	302	..	..	..
55	80	56	3	117	80	39	1	179	..	..	..	241	..	..	..	303	..	..	..
56	80	57	3	118	60	59	2	180	60	45	1	242	..	..	..	304	60	76	1
57	80	58	3	119	..	..	..	181	..	..	..	243	..	..	..	305	48	61	1
58	80	59	3	120	60	30	1	182	..	..	..	244	60	61	1	306	40	51	1
59	80	60	3	121	..	..	..	183	80	61	1	245	48	49	1	307	..	..	..
60	80	60	3	122	60	61	2	184	60	46	1	246	..	..	..	308	60	77	1
61	80	61	3	123	80	41	1	185	48	37	1	247	..	..	..	309	..	..	..
62	60	31	2	124	60	31	1	186	40	31	1	248	60	62	1	310	30	45	1
63	80	42	2	125	48	25	1	187	..	..	..	249	..	..	..	..	..	..	..
64	60	30	2	126	80	42	1	188	60	47	1	250	48	50	1	..	..	..	..
65	80	65	3	127	..	..	..	189	80	63	1	251	..	..	..	..	..	..	..
66	80	33	2	128	60	32	1	190	48	38	1	252	40	42	1	..	..	..	..
67	80	67	3	129	80	43	1	191	..	..	..	253	..	..	..	..	..	..	..
68	60	34	2	130	48	26	1	192	60	48	1	254	..	..	..	..	..	..	..
69	80	46	2	131	..	..	..	193	..	..	..	255	48	51	1	..	..	..	..
70	60	35	2	132	80	44	1	194	..	..	..	256	..	..	..	..	..	..	..
71	80	71	3	133	..	..	..	195	48	39	..	257	..	..	..	..	..	..	..

TABLE OF CHANGE WHEELS TO BE USED AT A WHEEL-CUTTING MACHINE, WITH A MANDRIL WHEEL OF 180 TEETH.

No. of Teeth to be cut.	Driver.	Driven.	No. of turns of Handle.	No. of Teeth to be cut.	Driver.	Driven.	No. of turns of Handle.	No. of Teeth to be cut.	Driver.	Driven.	No. of turns of Handle.	No. of Teeth to be cut.	Driver.	Driven.	No. of turns of Handle.
10	90	20	4	33	90	33	2	56	90	56	2	79	90	79	2
11	90	22	4	34	90	34	2	57	90	57	2	80	90	80	2
12	75	20	4	35	90	35	2	58	90	58	2	81	90	81	2
13	90	26	4	36	90	36	2	59	90	59	2	82	90	82	2
14	90	28	4	37	90	37	2	60	90	60	2	83	90	83	2
15	60	20	4	38	90	38	2	61	90	61	2	84	90	84	2
16	90	32	4	39	90	39	2	62	90	62	2	85	90	85	2
17	90	34	4	40	90	40	2	63	90	63	2	86	90	86	2
18	50	20	4	41	90	41	2	64	90	64	2	87	90	87	2
19	90	38	4	42	90	42	2	65	90	65	2	88	90	88	2
20	45	20	4	43	90	43	2	66	90	66	2	89	90	89	2
21	90	42	4	44	90	44	2	67	90	67	2	90	90	45	1
22	90	44	4	45	90	45	2	68	90	68	2	91	90	91	2
23	90	46	4	46	90	46	2	69	90	69	2	92	90	41	1
24	60	32	4	47	90	47	2	70	90	70	2	93	90	31	1
25	45	25	4	48	90	48	2	71	90	71	2	94	90	47	1
26	90	52	4	49	90	49	2	72	90	72	2	95	90	38	1
27	40	24	4	50	90	50	2	73	90	73	2	96	90	48	1
28	45	28	4	51	90	51	2	74	90	74	2	97	90	97	2
29	90	58	4	52	90	52	2	75	90	75	2	98	90	49	1
30	60	40	4	53	90	53	2	76	90	76	2	99	90	33	1
31	90	62	4	54	90	54	2	77	90	77	2	100	90	50	1
32	90	64	4	55	90	55	2	78	90	78	2	110	90	55	1

The *Driver* signifies, as I have before explained, the *Division Plate Wheel*, and *Driven*, the *Tangent Wheel*. The names of driver and driven saves much confusion.

SCREW CUTTING.

Screw Cutting is a general mechanical operation performed at a screw-cutting lathe, and of which the most important operation is the selection of the proper *Change*, or *Driving* and *Driven* Wheels. They are placed on the *Lathe Mandril*, *stud* (or *studs*), and *Leading Screw*, in their proper proportions, so as to cut any required pitch *of screw*.

A knowledge of this important branch of mechanical art is

necessary to every practical mechanic, but more especially so to turners. Various other valuable publications have appeared; but, nevertheless, I consider it my duty to contribute also my share of experience for the benefit of my fellow-workmen, being the result of practice and the careful study of this very important subject. It is to be hoped the rules laid down here are simple and sufficiently explicit to enable any one to find the proper driving and driven wheels without much difficulty—a knowledge of which will prove of material advantage, and be the means of saving the young mechanic many hours of valuable time.

The Screw-Cutting Lathe.

It is generally understood by intelligent mechanics what a Screw-Cutting Lathe is, and the uses to which it is put; that it has a set of "Change Wheels," numbering from 15 to 130 and 150 (Whitworth's), advancing by five teeth; and that, when placed on the lathe mandril, stud (or studs), and leading screw, are designated driving and driven wheels, the "driving" giving motion to the "driven" wheels.

On the Management of the Saddle of the Slide Lathe.

When the Saddle of the Slide Lathe has travelled up in cutting the screw to the right distance, the best way is to mark the top side of the top driving and bottom driven wheels with a piece of chalk, then draw the tool backwards by hand, leaving the leading screw in gear until the first time the wheels come right after the screw tool comes past the place of giving the cut; when so, place the poppet head against the saddle, give the second cut when the saddle has travelled the required distance, throw the nut out of gear, take the saddle back to the poppet-head, which will be right for starting again, and so continue until the screw is cut. This rule will answer for all ordinary even pitches; but in broken pitches, where it comes to an even number of teeth or threads in a long length, the poppet-head spindle would be standing out sometimes too far; in this case chalk the wheels before taking the cut, let the saddle travel the required distance, throw the nut out of gear, then take the saddle back against the poppet-head, and pull the lathe round until the chalk marks come to the top together, when the nut will be in position for dropping into gear again, and proceeding thus until the screw is cut.

When the number of threads per inch required to be cut can be divided by the number per inch of the leading screw, without any remainder, *the nut will* drop in any part of the leading screw.

In cutting double, triple, quadruple, or more threads, always place a wheel on the top that will divide into as many equal parts as threads to be cut, chalk the divisions on top wheel, and chalk one tooth on the second wheel, and when one thread is finished drop the quadrant and place the next division where the chalk mark appears on the second wheel, and so on until all the threads are finished.

On Speeding by Belts and Pulleys.

Rule.—Multiply the number of revolutions of the driving shaft per minute by all the drivers' diameters in inches and parts of inches, and multiply the diameters of all the driven together in inches and parts of inches, then divide the product of the drivers by the product of the driven, and the quotient will be the number of revolutions per minute.

General Rules for Change Wheels.

The best method to find the change wheels to cut any ordinary pitches (in inches and parts of inches), or any number of threads per inch, as may be required, is by the following general rule, which is at once simple, useful, and explicit:—

Place the number of threads per inch of the screw to be cut, and the number of threads per inch of the leading screw, in a *fractional form*—the number of threads per inch of the leading screw being represented by the *numerator*, and the number of threads per inch of the screw to be cut by the *denominator* (add cyphers), or multiply each by any one convenient number, and you will have the wheels required:—

Example.

Required the wheels to cut 8 threads per inch, the leading screw having 4 threads per inch, $\frac{40}{80}$ (cyphers added) gives 40 as the driver, and 80 the driven; or thus: $\frac{4}{8} \times 5 = \frac{20}{40}$, the wheels required.

Or, express the pitch to be cut in a fractional form, then by *multiplying* the *numerator* by the number of threads per inch of the *leading screw*, shews the proportion between the *driving and driven* wheels, as in the above example. If 4, 6, or 8 wheels are required, *increase* the proportions by adding cyphers, or multiplying by any convenient numbers. *Always select wheels* such that their number of teeth *multiplied together* equals the proportion of *the numerator* for the driving wheels, and the same for the denominator or driven wheels.

DECIMAL PITCHES.

At the present time a large number of engineering establishments, both in England and on the Continent, issue their drawings, or the greater part of them, in decimals, so that it frequently occurs that the pitches of screws are given in decimals. When the pitch of a screw is so given, it is not necessary to change it into its equivalent vulgar fraction. By attending to the following rule —for finding change wheels for decimal pitches—the method of computing will be very readily understood.

To find the change wheels to cut decimal pitches of all kinds, from five-thousandths of an inch pitch (·005), *or* 200 *threads per inch.*

Rule.—Mark down the given decimal or decimals for the numerator, and *below* them place 1, with as many *cyphers* as there are figures in the *numerator*, thus, $\frac{\cdot 005}{1000}$. Then *multiply* the *numerator* by the *number of threads* per inch in the *leading screw*, and the product will give the *proportion* of the *driving wheels* to that of the *driven*. The *numerator always* represents the *driving wheels*, and the *denominator* the *driven* wheels.

Examples.

Required to find 6 change wheels to cut ·005 (five-thousandths of an inch pitch), or 200 threads per inch, the leading screw having 4 threads per inch.

$$\frac{5}{1000} \times 4 = \frac{20}{1000} \times 20 = \frac{400}{20000} \times 20 = \frac{8000}{400000} = \frac{20 \times 20 \times 20}{100 \times 80 \times 50},$$

the wheels required.

Or, changed,

$$\left.\begin{array}{l} 15 \times 20 \times 20 \\ 30 \times 20 \times 20 \\ 20 \times 20 \times 20 \end{array}\right\}$$ Use the most convenient set of the three.

These sets are all in the same proportion.

$$\left\{\begin{array}{l} 100 \times 40 \times 75 \\ 100 \times 80 \times 75 \\ 100 \times 80 \times 50 \end{array}\right.$$

Required to find the change wheels to cut ·01 pitch, or 100 *threads per inch, the* leading screw having 4 threads per inch.

$$\frac{1 \times 4}{100} = \frac{400}{10000} = \begin{matrix} \text{or } \frac{15 \times 20}{20 \times 20} \\ \text{or } \frac{75 \times 100}{100 \times 100} \end{matrix}$$ the 2 sets required.

N.B.—It is necessary to state here, that *adding cyphers* to the *proportions* (or numerator and denominator) to *increase* the numbers, is a *quicker* method of procedure, and preferable to *multiplying*, and it answers the same purpose.

Required to find the change wheels to cut ·07 (seven-hundredths) of an inch pitch, the leading screw having 4 threads per inch.

$$\frac{7 \times 4}{100} = \frac{2800}{10000} = \begin{matrix} \text{or } \frac{35 \times 40}{70 \times 40} \\ \text{or } \frac{50 \times 100}{100 \times 100} \end{matrix}$$ either of the 2 sets may be used.

Required the wheels to cut ·05 (five-hundredths) of an inch pitch, leading screw having 4 threads per inch.

$$\frac{5 \times 4}{100} = \frac{20}{100},$$ the simple wheels required.

Required the wheels to cut ·08 (eight-hundredths) of an inch pitch, the leading screw having 4 threads per inch.

$$\frac{8 \times 4}{100} = \frac{3200}{10000} = \left.\begin{matrix} \frac{20 \times 80}{40 \times 80} \\ \frac{100 \times 100}{50 \times 100} \end{matrix}\right\}$$ the wheels required.

Required the wheels to cut ·09 (nine-hundredths) of an inch pitch, the leading screw having 4 threads per inch.

$$\frac{9 \times 4}{100} = \frac{3600}{10000} = \left.\begin{matrix} \frac{20 \times 90}{40 \times 90} \\ \frac{100 \times 100}{50 \times 100} \end{matrix}\right\}$$ the wheels required.

Required the wheels to cut ·10 (ten-hundredths), or one-tenth of an inch pitch, the leading screw having 4 threads per inch.

$$\frac{10 \times 4}{100} = \frac{40}{100},$$ the wheels required.

Required the wheels to cut ·20 (two-tenths) pitch, the leading *screw having* 4 threads per inch.

$$\frac{20 \times 4}{100} = \frac{80}{100},$$ the wheels required.

Required to find the wheels to cut ·25 (two and a half-tenths) pitch, the leading screw having 4 threads per inch.

$$\frac{25 \times 4}{100} = \frac{100}{100},$$ the wheels required.

Required to find the wheels to cut ·275 (two-tenths seven-hundredths and a half) pitch, the leading screw having 4 threads pitch.

$$\frac{275 \times 4}{1000} = \frac{1100}{1000} = \left.\begin{matrix} 44 \times 50 \\ \overline{22 \times 50} \\ \overline{20 \times 50} \\ 20 \times 100 \end{matrix}\right\} \text{wheels required; use either of the two sets.}$$

Required the wheels to cut ·70 (seven-tenths) pitch, the leading screw having 2 threads per inch.

$$\frac{70 \times 2}{100} = \frac{140}{100},$$ the wheels required.

Required the wheels to cut 90 (nine-tenths) pitch, the leading screw having 2 threads per inch.

$$\frac{90 \times 2}{100} = \frac{90}{\overline{180}}{\overline{100}}$$

50, the wheels required.

Required the wheels to cut a screw having 6 threads per inch, the leading screw having 2 threads per inch.

Leading screw $\frac{2}{6}$, add cyphers $= \frac{20}{60}$, or (Doubled.) $\frac{40}{120}$

Required the wheels to cut $5\frac{3}{4}$ threads per inch, the leading screw having 2 threads per inch.

$\frac{2}{5\frac{3}{4}}$: it is plain that in this example cyphers cannot be added until reduced to a simple fraction; therefore, we multiply the $5\frac{3}{4}$ by 4 (the denominator of the fraction), and the number of threads in the leading screw by the same number, thus:—

$$\frac{2 \times 4}{5\frac{3}{4} \times 4} = \frac{80}{230} = \frac{40}{115}$$

When the number of threads are given, and 4, 6, or 8 wheels are required.

Proceed according to former rule, reducing the simple fraction to a compound one, add cyphers as before. Then, any 2, 3, or 4 numbers (according to the number of wheels required), for a set, when multiplied together, they will equal the numerator (or driving wheels), and the denominator (or driven wheels), without any remainder, will be the wheels required.

Examples.

Required the wheels to cut 36 threads per inch, the leading screw having 4 threads per inch pitch.

$$\frac{4}{36} \text{ or } \frac{400}{3600} = \frac{20 \times 20}{40 \times 90} \text{ or } \frac{40 \times 20}{80 \times 90}$$

Required the wheels to cut $4\frac{1}{4}$ threads per inch, the leading screw having three threads per inch.

$$\frac{3}{4{\cdot}25} \times 4 = \frac{4 \times 3}{17} = \frac{12 \times 5}{17 \times 5} = \frac{60}{85} = \frac{120}{170}$$

Required the wheels to cut $14\frac{2}{3}$ threads per inch, the leading screw having 3 threads per inch.

$$14\tfrac{2}{3} \times 3 = 44 \text{ threads in 3 inches pitch.}$$

$$\frac{3 \times 3}{44} = \frac{900}{4400} = \frac{45 \times 20}{110 \times 40}, \text{ the wheels required.}$$

Required the wheels to cut 17 threads per inch, the leading screw having 3 threads per inch.

The pitch will be $\frac{1 \times 3}{17} = \frac{3}{17} \times 2 = \frac{600}{3400} = \frac{20 \times 30}{40 \times 85}$ multiplied by 2 (to increase the proportion), also 4 cyphers added to increase it still more, gives the number of wheels required.

Required the wheels to cut 6 threads per inch, the leading screw having two threads per inch.

$$\frac{1 \times 2}{6} = \frac{20}{6}, \text{ the wheels required.}$$

Required the wheels to cut 70 threads per inch, the leading screw *having 4 threads* per inch.

$$\frac{1 \times 4}{70} = \frac{400}{7000} = \frac{20 \times 25}{70 \times 100} = \frac{20 \times 20}{70 \times 125};$$ or not having 2 twenty wheels, add a quarter to twenty, which makes 25. Add also a quarter to 100, which makes 125, the proportion remaining still the same.

It is frequently required to find the change wheels to cut very rough pitches, or for rifling small guns in a lathe. In this case proceed exactly as before, when it will be found that the proportion of the numerator (or driving wheel) will be much larger for rifle pitches, and it must be represented by as many wheels as multiplied together will make the proportion without any remainder. Also the denominator or driven wheels in the same manner; when the proportions are not large enough, multiply them both by any convenient number, or add cyphers to them, if convenient.

Example.

Required the wheels to cut a pitch of 22·75 inches, the leading screw having 2 threads per inch.

$$\frac{22{\cdot}75}{1} \times 8 = \frac{182 \times 2}{8} = \frac{364000}{8000} = \frac{80 \times 70 \times 65}{20 \times 20 \times 20},$$ the wheels required.

$$\left\{\begin{array}{l} 80 \times 70 \times 130 \\ 60 \times 70 \times 130 \\ 60 \times 70 \times 65 \\ 120 \times 70 \times 65 \\ 100 \times 70 \times 65 \\ 80 \times 70 \times 65 \end{array}\right\} \left\{\begin{array}{l} 20 \times 20 \times 40 \\ 15 \times 20 \times 40 \\ 15 \times 20 \times 20 \\ 30 \times 20 \times 20 \\ 25 \times 20 \times 20 \\ 20 \times 20 \times 20 \end{array}\right\}$$

These six sets are all of the same proportions, and any of them, taking first of first set with first of second, and second with second, etc., will cut the exemplified pitch of 22·75.

When the number of threads per inch is given, and only two wheels are required, place the number of threads per inch contained in the leading screw for a numerator, and the number of threads per inch of the screw to be cut for a denominator, then add a cypher *to each, the required* wheels will be obtained.

To prove the wheels, and to find if they are in the same proportion as that of the leading screw, and screw to be cut.

Rule.—Multiply all the driving wheels into each other, and all the driven wheels into each other, divide the greater product by the less, then take the two numbers which represent the number of threads of the leading screw, and that of the screw to be cut; divide the greater by the less, and if the wheels are in their proper proportion, both quotients will be the same.

EXAMPLE.—From the wheels obtained, cut 22·75 pitch.

$\frac{80 \times 70 \times 65}{20 \times 20 \times 20} = \frac{364000}{8000} = 45{\cdot}5$, the quotient of the wheels.

$\frac{182}{4} = 45{\cdot}5$, the quotient of the leading screw and the screw to be cut; the two quotients being the same, *proves* that the change wheels are of of the proper proportions.

It may be useful to the learner to have the entire figures worked out, thus :—

65	20
80	20
5200	400
70	20
364000	8000
8)364·0(45·5	4)182·0(45·5
32	16
44	22
40	20
40	20
40	20

It sometimes occurs that at old screw-cutting lathes you have a leading screw with an odd pitch, as $\frac{5}{8}$, $\frac{3}{8}$, &c. When the pitch of a screw is expressed in this fractional form, the bottom figure indicates the number of threads, and the top one the number of inches; *that is,* $\frac{3}{8}$ *pitch* would be 8 threads in 3 inches; $\frac{5}{8}$ pitch would be

8 threads in 5 inches. Required the wheels to cut 6 threads per inch, the leading screw $\frac{3}{8}$ pitch.

$\frac{3}{8}$ pitch = 8 threads in 3 inches (of the leading screw); 6 threads per inch = 18 threads in 3 inches of the screw to be cut.

$\frac{8 \times 5}{18 \times 5} = \frac{40}{90}$, or thus: $\frac{40}{80}$, adding cyphers and halving are the wheels required.

Required the wheels to cut $\frac{3}{4}$ pitch, the leading screw having $\frac{3}{8}$ pitch.

As $\frac{3}{8}$ pitch = 8 threads in 3 inches (of leading screw), so is $\frac{3}{4}$ pitch = 4 threads in 3 inches (screw to be cut).

$$\frac{8 \times 10}{4 \times 10} = \frac{80}{40}$$

SQUARE THREADS.

A TABLE SHEWING THE NUMBER OF SQUARE THREADS PER INCH ON SCREWS, COMMENCING FROM $\frac{5}{16}$ TO $1\frac{1}{4}$ DIAMETER.

Diameter in inches and parts.	Number of threads per inch.	Diameter in inches and parts.	Number of threads per inch.
$\frac{5}{16}$	9	$\frac{7}{8}$	6
$\frac{3}{8}$	9	$\frac{15}{16}$	6
$\frac{7}{16}$	8	1	5
$\frac{1}{2}$	7	$1\frac{1}{8}$	4
$\frac{9}{16}$	7	$1\frac{1}{4}$	4
$\frac{5}{8}$	7	$1\frac{3}{8}$	3
$\frac{11}{16}$	7	$1\frac{1}{2}$	3
$\frac{3}{4}$	6	$1\frac{5}{8}$	$2\frac{3}{4}$
$\frac{13}{16}$	6	$1\frac{3}{4}$	$2\frac{1}{2}$

The following is the *rule* to obtain the *proper width* of the *screw-cutting tool*, to any ordinary number of square threads to be contained in one or more inches:—

Divide one or *more inches by the number of threads required to be cut* in that distance, and *half the quotient* is the width of the screw-cutting tool.

The above Rule is *simply to find the half of the pitch of the screw, which readily gives the required width*, as per the following diagram:

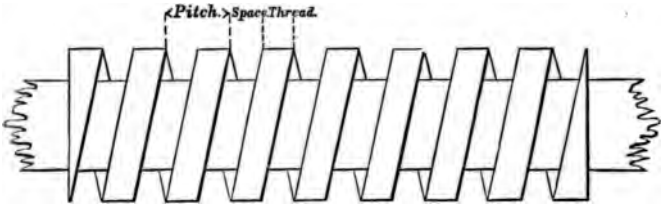


Fig. 65.

EXAMPLES.

(1.) Required the width of tool for $2\frac{1}{2}$ threads per inch.

$$2{\cdot}5)1{\cdot}00(\cdot4 \div 2 = \cdot2$$

100 two-tenths equal the width of tool.

(2.) Required the proper width of screw tool to cut $3\frac{3}{4}$ threads per inch.

$$3{\cdot}75)1{\cdot}000(\cdot266 \div 2 = \cdot133$$

750 } one tenth 3 hundredths
2500 } and 3 thousandths =
2250 } width of tool, nearly.
250

(3.) Required the width of screw-cutting tool to cut $8\frac{7}{8}$ threads per inch.

$$8{\cdot}875)1{\cdot}0000(\cdot112 \div 2 = \cdot056$$

8875 } 5 hundredths and
11250 } 6 thousandths =
8875 } thickness of tool, nearly.
23750
17750
6000

(4.) Required screw tool (width) to cut 6 threads per inch.

6)1·000(·166 ÷ 2 = ·083 = the thickness of tool, nearly.
6

40
36

40

SCREW-CUTTER'S GAUGE, AND GAUGE FOR GRINDING AND SETTING SCREW TOOLS.

Fig. 66.—GAUGE.

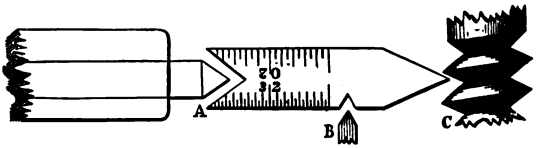


Fig. 67.

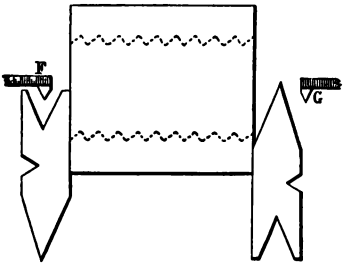


Fig. 68.—TOOL SET FOR CUTTING INSIDE THREADS.

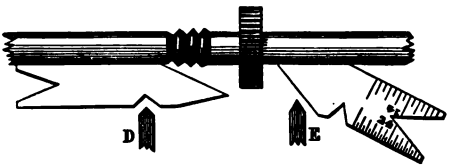


Fig. 69.

The above cuts shew the various uses to which the gauge can be put. In fig. 67, at A, is shewn the manner of gauging the angle to which a lathe centre should be turned; at B, the angle to which a screw thread-cutting tool should be ground, and at C, the correctness of the angle of a screw thread already cut. In fig. 69, the shaft with a screw thread is supposed to be held on the centre of a lathe. By applying the gauge, as shewn at D or E, the thread tool can be set at right angles to the shaft, and then fastened in place by the screw in tool post, thereby avoiding the imperfect leaning threads.

The angles used in this gauge are 55 degrees. The following parts to the inch can be determined by the four divisions upon the gauge (14, 20, 24, and 32)—viz., 2, 3, 4, 5, 6, 7, 8, 10, 14, 16, 20, 24, and 32.

METHOD OF DIVIDING ANY NUMBER OF DIVISIONS IN A SCREW-CUTTING LATHE, WITH CHANGE WHEELS.

To divide any given number of divisions in a lathe with change wheels, is simply to proportion the wheels to the number of divisions about to be divided, and the wheels are arranged as in the following diagram :—

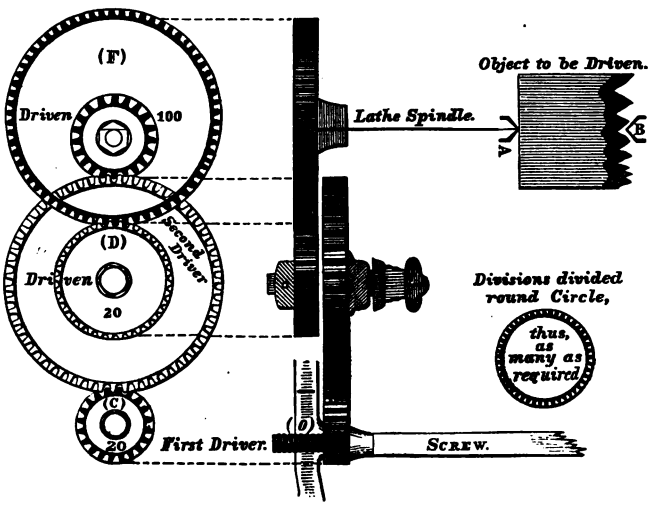


Fig. 70.

The work or cylinder to be divided is placed between the centres of the lathe (AB), and then we have the wheels named as in screw-cutting, drivers and driven; so that in dividing anything in a lathe the action is given from the screw wheel; therefore (C) is the first driving wheel, (D) the second, (E and F) the driven wheels. In dividing in the lathe the proportion is as so many to one, and if we are going to divide 350 divisions, it is as 350 to 1. As I have arranged my rule to move the first driver (C) one tooth, and then the following proportionate wheels give the number of divisions required.

Examples.

It is required to divide upon anything (circular) 375 divisions, in a lathe with the wheels used for screw-cutting. Here the proportion is as 375 to 1.

$$\frac{375}{1} \times 20 = \frac{7500}{20} = \frac{100 \times 75 \text{ driven.}}{20 \times 1 \text{ tooth, drivers.}}$$

This will seem strange at first: *the first driver being one tooth*, as a wheel cannot be made with one tooth only; but the division is *actually* divided by 3 wheels, not 4: the 1 means to *move* the first driver *one tooth*. It would not matter really what size or number of teeth the first driver was as long as you only moved it one tooth, as the calculation is made, as before mentioned, from 1 to 375 (or the moving of one tooth of the driver gives 375 divisions in the last driven wheel F), where the circle is being divided.

To divide 500 divisions on a circle in the lathe. Proportion is as 500 is to 1.

$$\frac{500}{1} \times 20 = \frac{10000}{20} = \frac{100 \times 100}{20 \times 1}$$

To divide 350 divisions on a circle in the lathe. Here the proportion is as 350 is to 1.

$$\frac{350}{1} \times 20 = \frac{7000}{20} = \frac{70 \times 100}{20 \times 1}$$

To divide 300 divisions on a circle in a lathe. Proportion is as *300 is to 1.*

$$\frac{300}{1} \times 20 = \frac{6000}{20} = \frac{60 \times 100}{20 \times 1}$$

The first numbers are multiplied by 20 merely to make the proportion larger, to be able to divide it with 2 suitable wheels. The first driving wheel in the sketch is intended to be 20. If required, a wheel with 100 teeth can be put on the spindle end (*o*) and 5 teeth taken for each division, as $5 \times 20 = 100$, and it would be easier to pull it round with.

CHANGE WHEELS REQUIRED FOR TURNING THE TAPERS ON GUNS, IN WHITWORTH'S 21-INCH CENTRES TAPER-TURNING LATHES.

To find the wheels to turn any tapers.

Change wheels, and the nearest tapers shewn above the guns, with the differences in small ends.

Rule.—Having the taper given, divide by 8, which gives proportion of wheels.

With Tapering Motion.

Taper.	Pinion on worm wheel (driver).	Wheel on stud (driver).	Pinion on stud (driver).	Wheel on cross shaft (driven).	Pinion on cross shaft (driven).	Wheel on screw (driven).
1 in 72	20	60	30	90	20	80
1 in 237·5	20	95	16	100	20	80
1 in 38·44	20	35	36	100	20	80
1 in 20·8	20	60	60	52	20	80

The above are the wheels given to turn tapers on guns, shewn in *the following* sketches :—

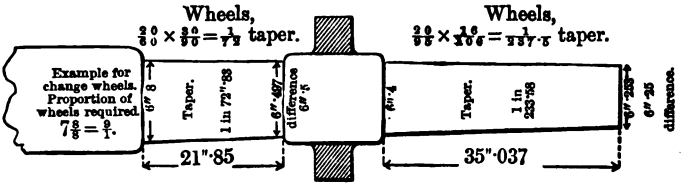


Fig. 71.

$$\frac{20}{60} \times \frac{30}{90} = \frac{1}{9}$$

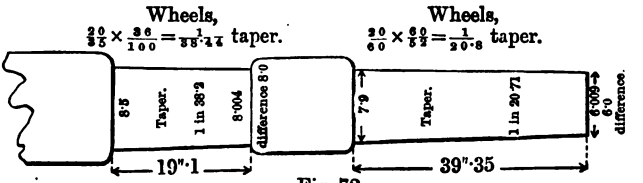


Fig. 72.

TABLE OF CHANGE WHEELS TO CUT THE UNIVERSAL GAS PIPE THREADS. LEADING SCREW HAVING TWO THREADS TO THE INCH.

No. of threads per inch.	Drivers.		Driven.		Remarks.
11·294	85	20	80	120	11·294 threads per inch is used for 1″·25, 1″·5, 1″·75, and 2 inch gas piping.
	20	85	120	80	
	15	85	80	90	
	25	85	80	150	
14	20	...	...	140	14 threads per inch is used for 1 inch, and also for ¾ gas piping.
	20	40	70	80	
	20	50	70	100	
	20	30	60	70	
	20	60	120	70	
18·412	30	20	65	85	Is used for ⅝ piping.
	20	30	85	65	
	40	30	85	130	
	15	40	65	85	
24	30	20	60	120	Used for small piping.
	40	20	80	120	
	20	40	80	120	
	20	20	40	120	
	20	45	120	90	

TABLE OF CHANGE WHEELS—*Continued.*

No. of threads per inch.	Drivers.		Driven.		Remarks.
11·294	85	30	80	120	Leading screw having three threads. Used for gas piping, as explained in the above remarks.
	30	85	80	120	
	25	85	80	100	
	15	85	80	60	
14	15	...	...	70	Do., do., do.
	30	...	...	140	
	30	40	70	80	
	30	45	70	90	
	30	...	...	140	
	25	30	50	70	
	30	50	70	100	
18·412	30	30	65	85	Do., do., do.
	20	45	65	85	
	30	40	80	120	
	30	60	85	130	
	20	90	85	130	
	15	60	85	65	
24	15	...	...	120	Do., do., do.
	30	40	80	120	
	20	30	80	60	
	25	30	60	100	
	20	30	40	120	
11·294.	20	40	47	100	Leading screws having four threads are used for gas piping, as explained in the remarks placed at the head of this table.
	25	85	80	150	
	20	85	80	120	
	25	85	100	120	
14	20	...	...	70	Do., do., do.,
	40	50	70	100	
	30	...	...	105	
	40	60	70	120	
18·412	40	30	65	85	Do., do., do.
	15	80	65	85	
	30	80	85	130	
	20	60	65	85	
24	20	...	...	120	Do., do., do.
	25	45	75	90	
	25	30	75	60	
	25	20	50	60	

WHITWORTH'S TABLE OF CHANGE WHEELS FOR 5, 6, AND 7-INCH PATENT SLIDE AND SCREWING LATHES, SHEWING THE PROPER WHEELS FOR CUTTING SCREWS OF VARIOUS PITCHES.

Leading Screw having 4 Threads per Inch.

Threads in 1 inch.	Wheel on mandril.	Wheel on guide.	Threads in 1 inch.	Wheel on mandril.	Wheel on stud.	Pinion.	Wheel on guide.	Threads in 1 inch.	Wheel on mandril.	Wheel on stud.	Pinion.	Wheel on guide.
1	80	20	12	40	...	...	120	39	40	90	15	65
1¼	80	25	13	20	...	...	65	40	40	100	20	80
1½	80	30	14	20	...	...	70	42	40	70	15	90
1¾	80	35	15	20	...	...	75	44	20	60	15	55
2	70	35	16	20	...	...	80	45	40	75	15	90
2¼	80	45	17	20	...	...	85	48	60	90	15	120
2½	80	50	18	20	...	...	90	50	40	75	15	100
2¾	80	55	19	20	...	...	95	51	40	85	15	90
3	80	60	20	20	...	...	100	52	20	60	15	65
3¼	80	65	21	20	...	...	105	55	40	75	15	110
3½	80	70	22	20	...	...	110	56	40	70	15	120
3¾	80	75	23	20	...	...	115	57	40	90	15	95
4	80	80	24	20	...	...	120	60	40	75	15	120
4¼	80	85	25	80	75	15	100	64	40	80	15	120
4½	80	90	26	60	65	15	90	65	20	75	15	65
4¾	80	95	27	40	45	15	90	66	40	90	15	110
5	80	100	28	60	70	15	90	72	40	90	15	120
5½	80	110	30	80	75	15	120	76	40	95	15	120
6	80	120	32	60	80	15	90	78	20	90	15	65
7	40	70	33	80	90	15	110	80	30	120	20	100
8	40	80	34	60	85	15	90	88	40	110	15	120
9	40	90	35	20	35	15	75	104	20	120	15	65
10	40	100	36	40	90	15	60	120	20	100	15	90
11	40	110	38	30	45	15	95	...	...	...	...	...

WHITWORTH'S TABLE OF CHANGE WHEELS FOR 8½ AND 10-INCH PATENT SLIDE AND SCREWING LATHES, SHEWING THE PROPER WHEELS FOR CUTTING SCREWS OF VARIOUS PITCHES.

Leading screw having two threads per inch.

Threads in 1 inch.	Wheel on mandril.	Do. guide.	Threads in 1 inch.	Wheel on mandril.	Do. stud.	Pinion.	Wheel on guide.	Threads in 1 inch.	Wheel on mandril.	Do. stud.	Pinion.	Wheel on guide.
1	80	40	8	40	30	15	80	45	20	75	15	90
1¼	80	50	9	80	60	15	90	48	30	90	15	120
1½	80	60	10	60	45	15	100	50	40	100	15	150
1¾	80	70	11	80	55	15	120	51	20	85	15	90
2	90	90	12	100	75	15	120	52	40	120	15	130
2¼	80	90	13	80	60	15	130	55	40	110	15	150
2½	80	100	14	60	70	15	90	56	50	140	15	150
2¾	80	110	15	80	90	15	100	57	20	90	15	112
3	80	120	16	60	80	15	90	60	40	120	15	150
3¼	40	65	17	60	85	15	90	64	25	80	15	150
3½	40	70	18	40	60	15	90	65	20	65	15	150
3¾	40	75	19	40	60	15	95	66	20	90	15	110
4	40	80	20	60	90	15	100	68	25	85	15	150
4¼	40	85	21	40	70	15	90	70	40	140	15	150
4½	40	90	22	60	90	15	110	72	20	90	15	120
4¾	40	95	24	40	80	15	90	75	20	75	15	150
5	40	100	25	40	75	15	100	76	25	95	15	150
5½	40	110	26	30	65	15	90	78	20	90	15	130
6	40	120	27	40	90	15	90	80	30	120	15	150
6½	40	130	28	30	70	15	90	84	20	90	15	140
7	40	140	30	30	75	15	90	85	20	85	15	150
...	...	...	32	30	80	15	90	88	20	110	15	120
...	...	...	33	40	90	15	110	90	20	90	15	150
...	...	...	34	30	85	15	90	95	20	95	15	150
...	...	...	35	40	75	15	140	96	25	125	15	150
...	...	...	36	40	90	15	120	100	20	100	15	150
...	...	...	38	30	90	15	95	104	25	130	15	150
...	...	...	39	40	90	15	130	110	20	110	15	150
...	...	...	40	30	90	15	100	112	25	140	15	150
...	...	...	42	40	90	15	140	120	20	120	15	150
...	...	...	44	30	90	15	110	130	20	130	15	150
...	...	...	...	...	...	...	...	140	20	140	15	150

ON RIFLING GUNS.

The spiral of any particular rifling is stated generally to be "one turn in so many inches or calibres," if uniform—*i.e.*, of one even twist from the commencement of the rifling to the muzzle; or, if an increasing spiral is to be adopted, it is said to be increasing from *o*, or so and so, at the breech, to one turn in so many inches (or calibres) at the muzzle. In developing any particular spiral—that is to say, supposing the interior surface of the bore laid upon a flat surface and opened out to view, the differences between the "uniform" and "increasing twist" become manifest; the former representing a straight line, and the latter a *curved* line, due to the particular elements of rifling embodied in the twist adopted. Two simple illustrations are only necessary to explain what has been stated. In the "uniform" twist the development is as follows:—

(*a*) = length of rifling in terms of inches or calibres due to one turn of the shot, measured in a line parallel to axis of gun; (*b*) = circumference of bore on the "1 turn," above mentioned; (*c*) = length of groove; and (*d*) = angle of groove with base, *a*. To illustrate this, take the 7 in. gun, whose twist is = 1 turn in 35 calibres.

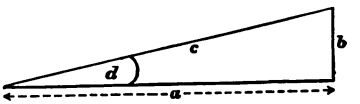


Fig. 73.

$\therefore a = 35 \times 7 = 245$ inches; $b = 7 \times 3{\cdot}1416 = 21{\cdot}99$ inches.

$C = \sqrt{a^2 + b^2} = \sqrt{60025 + 483{\cdot}55} = 246''$. Natural tangent $d = \frac{b}{a} = {\cdot}08976 = 5^\circ\ 4'$ (about).

Such are the proportions of the rifling when carried out to the extent given in the proposition; it now becomes an easy matter to determine the proportion any of the above results will take in the bore of the gun: for suppose l = length of rifling in bore, then, as $a : l :: b : x$, and so on—x representing the amount of twist in the length of the bore. In the case of the increasing spiral the statement is different, inasmuch as it is only at one point—viz., the muzzle—that the required amount of twist is given.

The other portions of the rifling diminish in the sharpness of their twist towards the breech, according to the formula adopted in the calculation.

In the guns of the English service the rifling is said to be of an "uniformly increasing twist," thereby meaning that if the shot *travels through a certain* unit of distance, and receives thereby a

certain amount of twist, in travelling through the next unit of distance it receives a similar amount of twist plus an increased amount due to curve adopted. Now, the only curve which embodies this principle is the parabola, and as such is therefore used in the service guns.

The mode of calculation is as follows :—

It must be borne in mind that one of the properties of the parabola is, that if a tangent be drawn from any point of the curve to the axis produced, and a co-ordinate drawn from that point to the axis, the abscissa thus produced is equal to the distance between *o* of the curve and the point at which the tangent cuts the axis produced. This being the case, we have first to determine the muzzle co-ordinate (*a*), or abscissa of the curve.

Fig. 74.

The dotted line (*b*) represents the tangent above mentioned, while (*c*) = the axis. For illustration, we will take the case of the 9-inch gun, which has an increasing twist from 0 to 1 turn in 45 calibres at muzzle—that is to say, that the angle (*d*) formed by the tangent (*b*) and base would represent the angle due to an uniform twist of 1 turn in 45 calibres. Such being the case, we have $9 \times 3{\cdot}1416 = 28{\cdot}2744$, and 45 calibres = 405 inches. The length of rifling in the gun is 107·5; therefore, as

$$
\begin{array}{l}
405 : 28{\cdot}2744 : : 107{\cdot}5 : \\
\qquad\quad 107{\cdot}5 \\
\hline
\qquad 1413720 \\
\qquad 1979208 \\
\qquad 2827440 \\
\hline
405)3039{\cdot}49800(7{\cdot}50 \\
\qquad 2835 \\
\hline
\qquad {\cdot}2044 \\
\qquad 2025 \\
\hline
\qquad\quad 199
\end{array}
\qquad \text{and } \frac{7{\cdot}5}{2} = a.
$$

The next step is to find the value of the co-ordinates of the

curve. The base (e) is subdivided into 10 equal parts, and from these points the co-ordinates are taken.

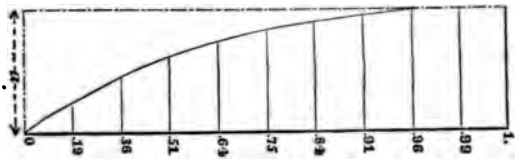


Fig. 75.

The value of the co-ordinates can be found by multiplying the abscissa (a), or in the case of the 9″-gun, 3·75, by the constant numbers shewn on the above diagram.

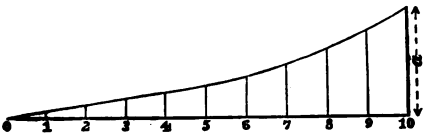


Fig. 76.

Receipt for Browning Guns.

1. The gun is to be well cleaned; when this is done give it a coating of lime to destroy all the grease, allow this coating to remain on the gun about 4 hours, then brush it off, and examine the gun to see if any oil or grease remains. If this one coating has not been sufficient to kill the grease, the operation must be repeated until it is removed.

2. After brushing it well, wipe it with a clean cloth. The gun is then to be coated with the browning mixture, which is laid on evenly by a piece of sponge, going over each part only once; this coating is allowed to remain on for about 24 hours, until quite dry and rusty. The rust is then to be scratched off by steel wire brushes; this being done, the gun is recoated with the browning mixture, and again allowed to stand until dry, when the scratching is repeated. This operation of recoating and scratching should be performed about 6 times, or sufficient to get a good permanent surface on the gun. When this is produced the gun should be placed in a cistern of boiling water, containing sufficient to cover the gun, the water being kept boiling with the gun in it for about *15 or 20 minutes.* The gun is then taken out and wiped dry with

clean cloths. If the appearance of the browning is not satisfactory after the boiling, the operation of coating with the mixture and scratching and boiling may be repeated until it is. In some cases it may be found desirable to go through the operations one, two, three, or four times.

3. After the gun is wiped dry, and the browning satisfactory, it is oiled over, and when rubbed off clean, it is finished.

Notes.—The gun should be allowed to become quite cool before applying the mixture, or a very great difficulty will be found in scratching off the rust. Putting the gun into boiling water stops the action of the acids, and prevents the rust from rising again. The bore of the gun and vent should be well plugged to prevent the acid and water from getting in. The gun will rust much better and quicker in a high temperature, say 80, 90, or 100 degrees Fahrenheit.

Browning Mixture.

Spirit of Sweet Nitre,.1 oz. Sulphate of Iron,.....1 oz. Tincture of Steel,.....2 oz. Nitric Acid,.....1 oz.	Mix.	Spirits of Salts,.....¼ oz. Corrosive Sublimate,.½ oz. Blue Vitriol,.....1 oz. Rain Water,.....1 quart

PART VI.—MISCELLANEOUS SUBJECTS.

The following subjects are variously classed under one head for the purpose of reference. The variety and usefulness of this classification will appear evident—the information given being important, and destined to facilitate the working of various difficult calculations.

Rules for determining by measurement the weight of round, square, and rectangular beams, bars, &c., of cast iron, &c.

A cubic foot of cast iron is equal to 1728 cubic inches, and also equal to 450 pounds (Avoirdupois weight). 1728 divided by 450 is equal to 3·84 cubic inches in one pound; hence the cubic inches in the solidity, divided by 3·84, will give the weight in pounds, or annex two cyphers to the solidity in inches, and divide by 384, the quotient will be the weight in pounds.

To find the number of cubic inches in one pound of any other body.

Divide the number of ounces in a cubic foot by 16, which gives the number of pounds; then divide 1728 by the number of pounds for the number of cubic inches in one pound.

To find the weight of square cast iron-bars.

Rule.—Square the side of the square in inches, multiply by the length in inches, annex two cyphers and divide by 384 for the weight in pounds, nearly. For wrought iron, divide by 355, and by 329 for cast brass.

EXAMPLE.

Required the weight of a cast-iron shaft 12 feet long by 6 inches square.

$6 \times 6 = 36$ and 12×12 feet $= 144$ inches, $144 \times 36 = 5184$ cubic inches, $\frac{518400}{384} = 1350$ pounds, or 12 hundredweights (112 lbs.) *and 6 pounds.*

To find the weight of flat cast iron.

Rule.—Multiply the breadth by the thickness and by the length in inches, annex two cyphers, and divide by 384 for the weight in pounds, nearly. For wrought iron divide by 355, and by 329 for cast brass.

EXAMPLE.

Required the weight of a bar of wrought iron $1\frac{1}{2}$ inches thick, by 6 inches broad, and 10 feet long.

$1\frac{1}{2} \times 6 = 9$, $9 \times 120 = 1080$ cubic inches, $\frac{108000}{355} = 304$ pounds.

To find the weights of round cast and wrought iron.

Rule.—Square the diameter in inches, multiply the square by the length in inches, annex two cyphers and divide by 489 for cast iron, by 452 for wrought iron, and by 416 for cast brass.

EXAMPLE.

Required the weight of a cast-iron shaft 12 feet, or 144 inches long, by 6 inches diameter.

$6 \times 6 = 36$ and $36 \times 144 = 5184$ cylindrical inches, $\frac{516400}{489} = 1060$ lbs., or 9 hundredweights 1 quarter and 24 pounds. The divisor 489 is found (by proportion) by making it as much greater as the area of the square is greater than the area of the circle.

Thus, .7854 : 1 :: 384 : 489.

To find the weight of a hollow cylinder of cast iron.

Rule.—Multiply the sum of the outside and inside diameters by their difference, and by the number of circular inches in the cylinder's length in inches. Annex two cyphers to the weight in pounds, nearly, by 452 for brass.

cylinder 72 inches long by 48 inches bore; $48 + 46 =$ $94 \times 2 = 188$

2768 lbs., or

A hollow ball.

two cyphers to the pounds, nearly.

Example.

Required the weight of a cast-iron ball, 12 inches diameter.
$12 \times 12 \times 12 = 1728$ spherical inches.
$\frac{172800}{733} = 235{\cdot}7$ pounds, or 2 hundredweights $11\frac{1}{2}$ pounds, nearly.

To find the weight of any cast-iron hollow ball.

Rule.—Cube the outside and inside diameters, subtract the less from the greater, annex two cyphers to the remainder, and divide by 733 for the weight in pounds, nearly.

Example.

Required the weight of a hollow cast-iron ball, the outside diameter being 12 inches, and the inside diameter being 10 inches.
$12 \times 12 \times 12 = 1728$, cube of 12. $10 \times 10 \times 10 = 1000$, cube of 10.
$1728 - 1000 = 278$, difference of their cubes.
$\frac{72800}{733} = 99$ pounds 5 ounces, or 3 quarters 15 pounds 5 ounces.
The divisor 733 is found by proportion. The solidity of the sphere is to the solidity of the cube as ·5236 is to 1. Thus,

·5236 : 1 : : 384 : 733.

Weight of Round and Square Cast Iron in pounds, one foot in length, from $\frac{1}{8}$ of an inch to 8 inches.

In pounds or parts of pounds Avoirdupois.

Inches.	Round.	Square.	Inches.	Round.	Square.	Inches.	Round.	Square.
	·046	·058	$2\frac{1}{8}$	15″·5	19″·53	$4\frac{7}{8}$	58″·34	74″·27
	·153	·195	$2\frac{1}{4}$	16″·91	21″·53	5	61″·35	78″·12
	·348	·442	$2\frac{3}{8}$	18″·52	23″·63	$5\frac{1}{8}$	64″·46	86″
	·613	·781	$2\frac{1}{2}$	20″·2	25″·83	$5\frac{1}{4}$	67″·64	82″·12
	·96	1″·22	3	22″·5	28″·13	$5\frac{3}{8}$	71″	90″·3
1	1″·38	1″·76	$3\frac{1}{8}$	24″	30″·52	$5\frac{1}{2}$	74″	94″·52
1	1″·88	2″·4	$3\frac{1}{4}$	26″	33″	$5\frac{5}{8}$	77″·64	98″·88
1	2″·45	3″·12	$3\frac{3}{8}$	28″	35″·6	$5\frac{3}{4}$	81″·12	103″·32
2	3″·10	98	$3\frac{1}{2}$	30″	38″·28	$5\frac{7}{8}$	84″·7	108″
2	3″·8	83	$3\frac{5}{8}$	32″·25	41″	6	88″·34	112″·5
2	4″·5	91	$3\frac{3}{4}$	34″·52	44″	$6\frac{1}{4}$	96″	122″
3	5″·3	63	$3\frac{7}{8}$	36″·85	47″	$6\frac{1}{2}$	104″	132″
3	6″·2	56	4	39″·24	50″	$6\frac{3}{4}$	112″	142″·4
3	7″·1	7	$4\frac{1}{8}$	41″·55	53″	7	120″	153″·12
3	8″·2	9	$4\frac{1}{4}$	44″·32	56″·44	$7\frac{1}{4}$	130″	164″
4	9″·4	0	$4\frac{3}{8}$	47″	59″·8	$7\frac{1}{2}$	138″	176″
4	10″·6	11	$4\frac{1}{2}$	49″·68	63″·28	$7\frac{3}{4}$	147″·4	188″
4	11″·9	32	$4\frac{5}{8}$	52″·5	66″·8	8	157″	200″
4	13″·3	53	$4\frac{3}{4}$	55″·36	70″·52			

Weight of Cast-Iron Balls, Diameter in Inches, and Weight in Pounds Avoirdupois, from 1 inch to 25 inches Diameter.

Diameter.	Weight.	Diameter.	Weight.	Diameter.	Weight.	Diameter.	Weight.	Diameter.	Weight.
1″	·136	4″⅞	15·8	8″¾	91·36	12″⅝	274·4	17″¾	760·2
1″⅛	·194	5″	17·05	8″⅞	95·66	12″¾	282·7	18″	795·6
1″¼	·266	5″⅛	18·364	9″	99·45	12″⅞	291	18″¼	829·2
1″⅜	·354	5″¼	19·739	9″⅛	103·65	13″	299·7	18″½	863·7
1″½	·46	5″⅜	21·182	9″¼	107·97	13″⅛	308·4	18″¾	899·2
1″⅝	·585	5″½	22·697	9″⅜	112·4	13″¼	317·3	19″	935·7
1″¾	·732	5″⅝	24·28	9″½	116·96	13″⅜	326·3	19″¼	973
1″⅞	·899	5″¾	25·927	9″⅝	121·64	13″½	335·6	19″½	1011·5
2″	1·091	5″⅞	27·664	9″¾	126·83	13″⅝	345	19″¾	1050·9
2″⅛	1·308	6″	29·468	9″⅞	131·38	13″¾	354·6	20″	1091·4
2″¼	1·554	6″⅛	31·348	10″	136·42	13″⅞	364·3	20″¼	1138·9
2″⅜	1·827	6″¼	33·3	10″⅛	141·69	14″	374·3	20″½	11·75
2″½	2·131	6″⅜	35·344	10″¼	146·79	14″⅛	384·4	20″¾	1218·8
2″⅝	2·467	6″½	37·465	10″⅜	152·27	14″¼	394·7	21″	1249·7
2″¾	2·837	6″⅝	39·67	10″½	157·92	14″⅜	405·2	21″¼	13·10
2″⅞	3·242	6″¾	41·935	10″⅝	163	14″½	415·9	21″½	1355·8
3″	3·683	6″⅞	44·33	10″¾	169·16	14″⅝	426·7	21″¾	1390·
3″⅛	4·163	7″	46·79	10″⅞	175·41	14″¾	437·7	22′	1452·6
3″¼	4·683	7″⅛	49·34	11″	181·59	14″⅞	449	22″¼	1502·7
3″⅜	5·244	7″¼	51·98	11″⅛	187·84	15″	460·4	22″½	1553·9
3″½	5·849	7″⅜	54·71	11″¼	193·86	15″¼	483·8	22″¾	1606·3
3″⅝	6·498	7″½	57·52	11″⅜	200·75	15″½	508	23″	1659·8
3″¾	7·195	7″⅝	60·47	11″½	207·48	15″¾	533	23″¼	1714·6
3″⅞	7·938	7″¾	63·48	11″⅝	214·29	16″	558·8	23″½	1770·5
4″	8·731	7″⅞	66·62	11″¾	221·31	16″¼	585·4	23″¾	1827·6
4″⅛	9·575	8″	69·84	11″⅞	228·37	16″½	612·8	24″	1884·6
4″¼	10·47	8″⅛	73·17	12″	235·74	16″¾	644	24″¼	1945·5
4″⅜	11·416	8″¼	76·6	12″⅛	243·15	17″	684	24″½	2006·3
4″½	12·43	8″⅜	80	12″¼	250·6	17″¼	700	24″¾	2068·3
4″⅝	13·5	8″½	83·78	12″⅜	258·5	17″½	731	25″	2131·6
4″¾	14·61	8″⅝	87·53	12″½	266·45	...	...	...	...

WEIGHT OF ROUND ROLLED IRON, FROM $\frac{1}{16}$ INCH TO 12 INCHES IN DIAMETER, 1 FOOT IN LENGTH.

Diam.	Weight.	Diam.	Weight.	Diam.	Weight.	Diam.	Weight.
inch.	lbs.	inch.	lbs.	inch.	lbs.	inch.	lbs.
$\frac{1}{16}$	·01	$\frac{1}{4}$	13·44	$\frac{5}{8}$	56·788	$\frac{1}{2}$	149·328
$\frac{1}{8}$	·041	$\frac{5}{16}$	14·975	$\frac{11}{16}$	59·9	$\frac{3}{4}$	159·456
$\frac{1}{16}$	·093	$\frac{3}{8}$	16·588	$\frac{3}{4}$	63·094	8·	169·856
$\frac{1}{4}$	·165	$\frac{7}{16}$	18·293	5·	66·35	$\frac{1}{4}$	180·696
$\frac{5}{16}$	·373	$\frac{1}{2}$	20·076	$\frac{1}{16}$	69·731	$\frac{1}{2}$	191·808
$\frac{1}{2}$	·663	$\frac{9}{16}$	21·944	$\frac{1}{4}$	73·172	$\frac{3}{4}$	203·26
$\frac{5}{16}$	1·043	3·	23·888	$\frac{5}{16}$	76·7	9·	215·04
$\frac{3}{4}$	1·493	$\frac{1}{16}$	25·926	$\frac{3}{8}$	80·304	$\frac{1}{4}$	227·152
$\frac{7}{16}$	2·032	$\frac{1}{4}$	28·04	$\frac{7}{16}$	84·001	$\frac{1}{2}$	239·6
1·	2·654	$\frac{5}{16}$	30·24	$\frac{1}{2}$	87·776	$\frac{3}{4}$	252·376
$\frac{1}{16}$	3·359	$\frac{3}{8}$	32·512	$\frac{9}{16}$	91·634	10·	265·4
$\frac{1}{4}$	4·147	$\frac{7}{16}$	34·886	6·	95·552	$\frac{1}{4}$	278·924
$\frac{5}{16}$	5·019	$\frac{1}{2}$	37·332	$\frac{1}{16}$	103·704	$\frac{1}{2}$	292·688
$\frac{1}{2}$	5·972	$\frac{9}{16}$	39·864	$\frac{1}{8}$	107·86	$\frac{3}{4}$	306·8
$\frac{5}{16}$	7·01	4·	42·464	$\frac{1}{2}$	112·16	11·	321·216
$\frac{3}{4}$	8·128	$\frac{1}{16}$	45·174	$\frac{5}{16}$	116·484	$\frac{1}{4}$	336·004
$\frac{7}{8}$	9·333	$\frac{1}{8}$	47·952	$\frac{3}{4}$	120·96	$\frac{1}{2}$	351·104
2·	10·616	$\frac{3}{16}$	50·815	7·	130·048	$\frac{3}{4}$	366·566
$\frac{1}{8}$	11·988	$\frac{1}{2}$	53·76	$\frac{1}{4}$	139·544	12·	382·208

Computation.—Ascertain by the rules in mensuration the number of cubic inches the piece of iron contains, and multiply the same by ·2816, which equals the weight of a cubic inch.

DIVISORS FOR CAST IRON.

3·84 cubic inches in one pound.
4·89 circular inches in one pound.
7·33 spherical inches in one pound.

DIVISORS FOR WROUGHT IRON.

3·55 cubic inches in one pound.
4·52 circular inches in one pound.

DIVISORS FOR CAST BRASS.

3·29 cubic inches in one pound.
4·16 circular inches in one pound.

WHITWORTH'S SYSTEM OF BOLT AND NUT HEADS: DIMENSIONS, SIZES ACROSS THE FLATS, FROM $\frac{1}{4}$ TO 2 INCHES IN DIAMETER OF BOLT.

$\frac{1}{4}$ =	·5″	$1\frac{1}{8}$ =	1″·85
$\frac{5}{16}$ =	·7″	$1\frac{1}{4}$ =	2″·05
$\frac{3}{8}$ =	·7″	$1\frac{3}{8}$ =	2″·2
$\frac{7}{16}$ =	·8″	$1\frac{1}{2}$ =	2″·4
$\frac{1}{2}$ =	·9″	$1\frac{5}{8}$ =	2″·5
$\frac{5}{8}$ =	1″·1	$1\frac{3}{4}$ =	2″·7
$\frac{3}{4}$ =	1″·3	$1\frac{7}{8}$ =	2″·9
$\frac{7}{8}$ =	1″·45	2″ =	3″·1
1″ =	1″·65		

WHITWORTH'S BOLT AND NUT HEAD.

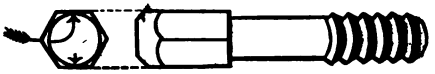


Fig. 77.

The spanners (screw-keys) of these, as all other sizes of bolts, should be left about ·020 slack on the jaws, thus—

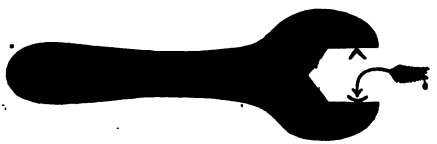


Fig. 78.

2-hundredths larger than dimension shewn on preceding table.

CO-EFFICIENTS OF AXLE FRICTION.

Substances.	Coefficient of frictional action.
Wrought iron on bell metal, ...	·075, with oil, tallow.
Cast iron on bell metal, ...	·075 ,, ,,
Wrought iron on cast iron, ...	·066 ,, ,,
Cast iron on cast iron, ...	·075 ,, ,,
Cast iron on lignum-vitæ, ...	·1 ,, ,,
Lignum-vitæ on cast iron, ...	·116 ,, ,,
Lignum-vitæ on lignum-vitæ, ...	·07 ,, ,,
Wrought iron on cast iron, ...	·054 ,, ,,
Wrought iron on lignum-vitæ, ...	·125 ,, ,,

WHITWORTH'S SIZES OF TAPS, FROM $\frac{1}{8}$ TO 2 INCHES DIAMETER.

DIMENSIONS.

Diameter of tap.	Full length.	Length of screwed part.	Diameter of bottom of thread.	Length of square head.	No. of threads per inch.	Size of square head.
$\frac{1}{8}$	1″·8	·75″	·16″	·3″	40	·1″
$\frac{3}{16}$	2 ·15	1 ·1	·19″	·3″	24	·15″
$\frac{1}{4}$	2″·6	1″·2	·26″	·35″	20	·2″
$\frac{5}{16}$	3″	1″·5	·23″	·4″	18	·2″
$\frac{3}{8}$	3″·25	1″·625	·297″	·4″	16	·25″
$\frac{7}{16}$	3″·65	1″·9	·34″	·45″	14	·28″
$\frac{1}{2}$	3″·95	2″·15	·38″	·55″	12	·3″
$\frac{5}{8}$	4″·15	2″·4	·5″	·6″	11	·4″
$\frac{3}{4}$	4″·7	2″·6	·595″	·75″	10	·45″
$\frac{7}{8}$	5 ·1	3″	·72″	·8″	9	·55″
1″	5″·6	3 ·3	·84″	·8″	8	·6″
1″$\frac{1}{8}$	6″·2	3″·65	·93″	·95″	7	·65″
1″$\frac{1}{4}$	6″·6	3″·85	1″·06	1″·15	7	·8″
1″$\frac{3}{8}$	7″·4	4″·45	1″·14	1″·15	6	·8″
1″$\frac{1}{2}$	8″·1	4″.9	1″·24	1″·15	6	·9″
1″$\frac{5}{8}$	8″·85	5″·45	1″·35	1″·25	5	1″·1
1″$\frac{3}{4}$	9″·6	5″·8	1″·48	1″·4	5	1″·1
1″$\frac{7}{8}$	10″·5	6″·4	1″·575	1″·6	4·5	1″·2
2″	11″·4	6″·9	1″·7	1″·75	4·5	1″·2

THE WEDGE.

The wedge is a double inclined plane.

To compute the power of the wedge, when two bodies or two parts of a body are forced, or sustained in a direction parallel to the back of the wedge.

Rule.—Multiply the weight or resistance to be sustained by half the depth of the back of the wedge, and divide by the length.

Or, $\frac{W \times D \div 2}{L} = P$, D representing the depth of the back, and L the length.

EXAMPLE.—The depth of the back of a double wedge (faced) is 6 inches, and the length of it through the middle 10 inches, what is the power necessary to sustain or overcome a resistance of 150 lbs. ?

$$\frac{150 \times 6 \div 2}{10} = \frac{450}{10} = 45 \text{ lbs.}$$

THE SCREW.

A screw is a revolving inclined plane.

To compute the length and height of the plane of a screw.

The screw being an inclined plane wound around a cylinder, the length of the plane is ascertained by adding the square of the circumference of the screw to the square of the distance between the threads, and taking the square root of the sum; and the height or pitch of the screw is the distance between its consecutive threads.

To compute the power of the screw.

Rule.—Multiply the weight or resistance to be sustained by the pitch of the threads, and divide the product by the length of the plane; or, $\frac{w \times p}{l} = P$, p representing the pitch.

EXAMPLE.—What is the power requisite to raise a weight of 8000 lbs. by a screw of 12 inches circumference and 1 inch pitch ?

$12^2 + 1^2 = 145$, and $\sqrt{145} = 12{\cdot}04159$. Then, $\frac{8000 \times 1}{12{\cdot}0416} = 664{\cdot}36$ lbs.

To compute the weight of a cast-iron shot from its diameter.

A cast-iron shot of 4 inches diameter weighs 8·736 lbs. Therefore, $\frac{8{\cdot}736}{64} = {\cdot}1364$ of the cube of the diameter is the weight of a shot of any diameter, for the weights of spheres are as the cubes of their diameters.

Rule.—Multiply the cube of the diameter in inches by ·1364, and the product is the weight.

EXAMPLE.—What is the weight of a cast-iron shot 10 inches diameter?

$$10^3 \times \cdot1364 = 136\cdot4 \text{ lbs.}$$

To compute the diameter of a shot from the weight.

Rule.—Divide the weight in pounds by ·1364, and the cube root * of the quotient is the diameter.

EXAMPLE.—What is the diameter of a cast-iron shot, its weight being 99·5 lbs.?

$$99\cdot5 \div \cdot1364 = 729 \text{ and } \sqrt[3]{729} = 9 \text{ inches.}$$

To compute the weight or diameter of a leaden shot.

N.B.—A lead shot 4 inches in diameter weighs 13·744 lbs. Therefore, $\frac{13\cdot744}{64} = \cdot12147$ of the cube of the diameter is the weight of a shot of any diameter.

Rule.—Multiply the cube of the diameter in inches by ·2147, and the product is the weight; or divide the weight in pounds by ·2147, and the cube root of the quotient is the diameter.

EXAMPLE.—What is the weight of a lead shot 10 inches in diameter?

$$10^3 \times \cdot2147 = 214\cdot7 \text{ lbs.}—\textit{Ans.}$$

TABLE OF CONTRACTION AND EXPANSION IN COOLING AND HEATING.

Names of bodies.	Melting and boiling points.	Contracts in cooling in parts of an inch for each foot in length.	Expands in heating from 32 to 212 degrees of Fahrenheit, the length being 1·00000.	Power of conducting heat.
Cast-iron melts ...	...	·124	·00111	1·2
Wrought-iron, welding hot...	...	·137	·00122	1·1
Copper melts	...	·193	·00172	1·0
Brass melts	...	·210	·00187	1·0
Steel hot (red).....	1077°	·133	·00118	1·0
Zinc melts	700°	·329	·00294	1·0
Mercury boils	660°	...	·01851	...
Lead melts	594°	·319	·00286	2·5
Bismuth melts.....	476°	·156	·00139	...
Tin melts.....	442°	·278	·00248	1·7
Water boils.....	212°	...	·04002	...

* *See Appendix Tables* for all cubes, squares, &c., and other information.

TABLE OF GAS TAPS, IN FRACTIONAL PARTS OF AN INCH.

Size of tube.	Diameter of tap.	Length of tap.	Length of screwed part.	Length of square head.	Size of bottom of screw or tapping hole.	Number of threads per inch.	Wheels to cut them, 4 threads per inch of leading screw wheels.			
Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Drivers.		Driven.	
$\frac{1}{8}$	$\frac{3}{8}$ $\frac{1}{32}$	$2\frac{1}{2}$	1	$\frac{7}{16}$	$\frac{5}{16}$ $\frac{3}{64}$	28	20	75	70	75
$\frac{1}{4}$	$\frac{1}{2}$ $\frac{1}{32}$	$2\frac{3}{4}$	$1\frac{1}{8}$	$\frac{1}{2}$	$\frac{7}{16}$	19	40	45	90	95
$\frac{3}{8}$	$\frac{5}{8}$ $\frac{1}{32}$	3	$1\frac{3}{4}$	$\frac{5}{8}$	$\frac{9}{16}$	19	40	45	90	95
$\frac{1}{2}$	$\frac{3}{4}$ $\frac{1}{16}$	$3\frac{1}{2}$	2	$\frac{3}{4}$	$\frac{5}{8}$ $\frac{3}{32}$	14	20	...	...	70
$\frac{5}{8}$	$\frac{7}{8}$ $\frac{1}{32}$	4	$2\frac{1}{4}$	$\frac{7}{8}$	$\frac{3}{4}$ $\frac{1}{32}$	14	20	...	...	70
$\frac{3}{4}$	1 $\frac{1}{32}$	$4\frac{1}{2}$	$2\frac{1}{2}$	1	$\frac{7}{8}$ $\frac{1}{32}$	14	20	...	...	70
1	$1\frac{1}{4}$ $\frac{1}{32}$	5	$3\frac{1}{4}$	$1\frac{1}{8}$	$1\frac{1}{8}$ $\frac{1}{32}$	11	20	...	...	55
$1\frac{1}{4}$	$1\frac{5}{8}$ $\frac{1}{32}$	$5\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{2}$ $\frac{1}{32}$	11	20	...	...	55
$1\frac{1}{2}$	$1\frac{7}{8}$ ...	6	$3\frac{3}{4}$	$1\frac{3}{8}$	$1\frac{3}{4}$ ...	11	20	...	...	55
$1\frac{3}{4}$	$2\frac{1}{32}$ ...	$6\frac{1}{2}$	4	$1\frac{1}{2}$	$1\frac{7}{8}$ $\frac{1}{32}$	11	20	...	...	55
2	$2\frac{1}{16}$ $\frac{1}{32}$	$6\frac{3}{4}$	4	$1\frac{1}{2}$	$2\frac{3}{16}$ $\frac{1}{32}$	11	20	...	...	55

TABLE OF GAS TAPS FOR GAS PIPING, IN DECIMAL PARTS OF INCHES.

Size of tube.	Diameter of tap.	Length of tap.	Length of screwed part.	Length of square head.	Size of the bottom of screw or tapping hole.	Number of threads per inch.	Whitworth's threads, wheels for 4 threads per inch of leading screw.			
Part of Inches.	Parts of Inches.	Parts of Inches.	Parts of Inches.	Parts of Inches.	Parts of Inches.	Threads	Drivers.		Driven.	
·125	·4062	2″·5	1″	·4375	·3593	28	20	75	70	75
·25	·5312	2″·25	1″·125	·5	·4375	19	40	45	90	95
·375	·6562	3″	1″·75	·625	·5625	19	40	45	90	95
·5	·8125	3″·5	2″	·75	·718	14	20	...	...	70
·625	·9062	4″	2″·5	·875	·7812	14	20	...	...	70
·75	1″·0312	4″·5	2″·5	1″	·9062	14	20	...	...	70
1″	1″·2812	5″	3″	1″·125	1″·1562	11	20	...	...	55
1″·25	1″·6562	5″·5	3″·25	1″·25	1″·5312	11	20	...	...	55
1″·5	1″·875	6″	3″·5	1″·5	1″·75	11	20	...	...	55
1″·75	*2″·0312*	6″·5	3″·75	1″·5	1″·9062	11	20	...	...	55
2″	*2″·3437*	7″	4″	1″·5	2″·2187	11	20	...	...	55

WEIGHT AND VOLUMES OF CAST IRON AND LEAD BALL, FROM 1 TO 20 INCHES DIAMETER.

Diameter.	Volume.	Cast Iron.	Lead.
Inches.	Cubic Inches.	Pounds.	Pounds.
1	·5235	·1365	·2147
1½	1·7671	·4607	·7248
2	4·1887	1·092	1·718
2½	8·1812	2·1328	3·3554
3	14·1371	3·6855	5·7982
3½	22·4492	5·8525	9·2073
4	33·5103	8·7361	13·744
4½	47·7129	12·4387	19·569
5	65·4498	17·0628	26·843
5½	87·1137	22·7206	35·729
6	113·0973	29·4845	46·385
6½	143·7932	37·4528	58·976
7	179·5943	46·8203	73·659
7½	220·8932	57·587	90·598
8	268·0825	69·8892	109·952
8½	321·555	83·8396	131·883
9	381·7034	99·5103	156·553
9½	448·9204	117·0338	184·121
10	523·5987	136·5025	214·749
11	696·9098	181·7648	285·832
12	904·7784	235·8763	371·096
13	1150·346	299·623	471·806
14	1436·754	374·5629	589·273
15	1767·145	460·6959	724·781
16	2144·66	559·1142	876·616
17	2572·44	670·7168	1055·066
18	3053·627	796·0825	1252·422
19	3591·363	936·2708	1472·97
20	4188·79	1092·02	1717·995

To compute the weight of copper.

Rule.—Ascertain the number of cubic inches in the piece, multiply them by ·32118 (see Rules of Specific Gravity), and the product will give the weight in pounds.

EXAMPLE.

What *is the weight* of a copper plate ½ an inch in thickness by *16 inches square?*

$16^2 = 256$
$\cdot 5 = \frac{1}{2}$ an inch

$128 \times .32118 = 41 \cdot 111$ pounds.

To compute the weight of lead.

Rule.—Ascertain the number of cubic inches in the piece, and multiply the sum by ·41015 (see Rules of Specific Gravity), and the product will give the weight in pounds.

EXAMPLE.

What is the weight of a leaden pipe 12 feet long, 3·75 inches diameter, and 1 inch thick?

Notice.—By rules in mensuration of surfaces, to ascertain the area of cylindrical rings.

Area of $(3 \cdot 75 + 1 + 1) = 25 \cdot 967$
Area of $3 \cdot 75 = 11 \cdot 044$

$14 \cdot 923$ area of ring.
$144 = 12$ feet.

$2148 \cdot 912 \times 41015 = 881 \cdot 376$ pounds.

To compute the weight of ordinary brass castings.

Rule.—Ascertain the number of cubic inches in the piece, multiply them by ·3112 (see Rules of Specific Gravity), and the product will give the weight in pounds.

SPECIFIC GRAVITIES.

The specific gravity of a body is the proportion it bears to the weight of another body of *known* density.

If a body float on a fluid, the part immersed is to the whole body as the specific gravity of the body is to the specific gravity of the fluid.

When a body is immersed in a fluid, it loses such a portion of its own weight as is equal to that of the fluid it displaces.

An immersed body, ascending or descending in a fluid, has a force equal to the difference between its own weight and the *weight of its* bulk of the fluid, less the resistance of the fluid to its *passage.*

Water is well adapted for the standard of gravity; and as a cubic foot of it weighs 1000 ounces avoirdupois, its weight is taken as the unit—viz., 1000.

To Ascertain the Specific Gravity of a Body *Heavier* than Water.

Rule.—Weigh it both in and out of water, and note the difference; then, as the weight lost in water is to the whole weight, so is 1000 to the specific gravity of the body. Or, $\frac{W \times 1000}{W - w} = G$, *w representing the weight in water, and* G *the specific gravity.*

Example.

What is the specific gravity of a stone which weighs in air 15 lbs., in water 10 lbs.?

$15 - 10 = 5$; then $5 : 15 :: 1000 : 3000$ *spec. grav.*

To Ascertain the Specific Gravity of a Body *Lighter* than Water.

Rule.—Annex to the lighter body another that is heavier than water or the fluid used; weigh the piece added and the compound mass separately, both in and out of water or the fluid; ascertain how much each loses in water or the fluid, by subtracting its weight in water or the fluid from its weight in air, and subtract the less of these differences from the greater; then,

As the last remainder is to the weight of the light body in air, so is 1000 to the specific gravity of the body.

Example.

What is the specific gravity of a piece of wood that weighs 20 lbs. in air; annexed to it is a piece of metal that weighs 24 lbs. in air and 21 lbs. in water, and the two pieces in water weigh 8 lbs.?

$20 + 24 = 44 - 8 = 36$ *loss of compound mass in water;*
$24 - 21 = 3 =$ *loss of heavy body in water,*

$33 : 30 :: 1000 : 606 =$ *spec. grav.*

To Ascertain the Specific Gravity of a Fluid.

Rule.—Take a body of known specific gravity, weigh it in and out of the fluid; then as the weight of the body is to the loss of *weight, so is the specific* gravity of the body to that of the fluid.

EXAMPLE.

What is the specific gravity of a fluid in which a piece of copper (*spec. grav.* = 9000) weighs 70 lbs. in and 80 lbs. out of water?

$$80 : 80 - 70 = 10 : 9000 : 1125 \textit{ spec. grav.}$$

IRON—ITS PURPOSES AND PROPERTIES.

The foreign substances which iron contains modify its essential properties. Carbon adds to its hardness, but destroys some of its qualities, and produces cast iron or steel according to the proportion it contains. Sulphur renders it fusible, difficult to weld, and brittle when heated or "hot short." Phosphorus renders it "cold short," but may be present in the proportion of $\frac{2}{1000}$ to $\frac{3}{1000}$ without affecting injuriously its tenacity. Antimony, arsenic, and copper have the same effect as sulphur, the last in a greater degree.

CAST IRON.

The process of making cast iron depends much upon the description of fuel used, whether charcoal, coke, bituminous or anthracite coals. A larger yield from the same furnace, and a greater economy in fuel, are effected by the use of a hot blast. The greater heat thus produced causes the iron to combine with a larger percentage of foreign substances.

Cast iron for purposes requiring great strength should be smelted with a cold blast. Pig iron, according to the proportion of carbon which it contains, is divided into foundry iron and forge iron, the latter adapted only to conversion into malleable iron; while the former, containing the largest proportion of carbon, can be used either for casting or bars.

There are many varieties of cast iron, differing by almost insensible shades; the two principal divisions are gray and white, so termed from the colour of their fracture. The properties are very different.

Gray iron is softer and less brittle than white iron; it is in a slight degree malleable and flexible, and is not sonorous; it can be easily drilled and turned in a lathe, and does not resist the file It has a brilliant fracture, of a gray or sometimes a bluish-gray colour; the colour is lighter as the grain becomes closer, and its hardness increases at the same time. It melts at a lower heat than white iron, and preserves its fluidity longer. The colour of the fluid metal is red, and deeper in proportion as the heat is lower; it does not

adhere to the ladle; it fills the moulds well, contracts less, and contains fewer cavities than white iron; the edgings of its castings are sharp, and the surfaces smooth and convex. A medium-sized grain, bright gray colour, fracture sharp to the touch, and a close compact texture indicate a good quality of iron. A grain either very large or very small, a dull earthy aspect, loose texture, dissimilar crystals mixed together, indicate an inferior quality.

Gray iron is used for machinery and ordnance purposes where the pieces are to be bored or fitted. Its tenacity and specific gravity are diminished by annealing. Its mean specific gravity is 7·2.

White iron is very brittle and sonorous; it resists the file and the chisel, and is susceptible of high polish; the surface of its castings is concave; the fracture presents a silvery appearance, generally fine grained. Its mean specific gravity is 7·5.

Mottled iron is a mixture of white and gray; its castings have a plane surface, with edges slightly rounded. It is suitable for shot, shell, &c.

The colour and texture of cast iron depend greatly upon the volume of the casting and the rapidity of its cooling; a small casting, which cools quickly, is almost always white, and the surface of large castings partakes more of the qualities of white metal than the interior.

Wrought Iron.

Wrought iron is made from the pig iron in a bloomery fire or in a puddling furnace—generally in the latter. The process consists in melting it and keeping it exposed to a great heat, constantly stirring the mass, bringing every part of it under the action of the flame until it loses its remaining carbon, when it becomes malleable iron. When, however, it is desired to obtain iron of the best quality, the pig iron should be refined.

The puddling furnace is a reverberatory furnace, where the flame of bituminous coal is brought to act directly on the metal. The metal is first melted; the puddler then stirs it, exposing each portion in turn to the action of the flame, and continues this as long as he is able to work it. When it has lost its fluidity, he forms it into balls, weighing from 80 to 100 lbs., which are next passed to the shingler.

The bloomery resembles a large fire, where charcoal and a strong blast are used; and the refined metal or the pig iron, after being broken into pieces of the proper size, is placed before the blast, directly in contact with charcoal; as the metal fuses, it falls into a cavity left for the purpose below the blast, when the bloomer *works it into the shape* of a ball, which he places again before the

blast, with fresh charcoal; this operation is generally repeated, when the ball is ready for the shingler, as above.

Iron is improved in its several processes by judicious working, reheating it, and hammering or rolling it: other things being equal, the best iron is that which has been wrought most.

STEEL—ITS PROPERTIES AND GENERAL USES IN MECHANICS.

The varieties of steel are very numerous, the principal ones being as follows:—

Natural steel, Blister, Tilted, Shear, Cast, Fancy (or alloys), and Indian steel, termed Wootz.

Natural steel is obtained by reducing rich and pure descriptions of iron ore with charcoal, and refining the cast iron so as to deprive it of a sufficient portion of carbon to bring it to a malleable state.

After being tempered, steel is not easily broken; it welds rapidly, it does not crack or split, it bears a very high heat, and preserves the capability of hardening after repeated working.

The process of hardening requires great attention, and the loosening of the oxide scales from the surface must be attended to particularly, as the degree of hardness materially depends upon the heat and the rapidity of cooling. Steel will thus be made hard enough to resist the action of the hardest files, and becomes essentially brittle. It must be understood that the degree of heat and the temperature, and cooling medium, must be chosen with reference to the quality of the steel and the purposes for which it is intended. Avoid oils and other fatty substances in the process of cooling, for the lower the heat the steel becomes harder and better.

Letting down the temper, as it is termed, is performed by heating the hardened steel to a certain degree and cooling it quickly. To accurately ascertain this process, simply watch the surface of the steel from the film of oxide formed in the process of hardening. The degrees of heat to which the several colours correspond are as follows:—

At 430°, a very fine yellow, for hammer faces, drills, &c.
At 450°, a pale straw colour, for hammer faces, drills, &c.
At 470°, a full yellow, for shears, scissors, turning tools, &c.
At 490°, a brown colour, for shears, scissors, turning tools, &c.
At 510°, brown with purple spots, for plane-irons, knives, &c.
At 538°, purple, for plane-irons, knives, &c.

At 550°, dark blue, for cold chisels, axes, cutlery, &c.
At 560°, full blue, for cold chisels, axes, cutlery, &c.
At 600°, grayish blue, verging upon black, for saws, sword blades, &c.

Annealing Cast Iron.

In the process of annealing cast iron, it is not desirable that the metal should be brought to any more than a red heat, as otherwise the smaller pieces and their bars might not only bend but even melt. When the iron has become red hot in a charcoal fire, it should be kept completely covered, and the fire allowed to go gradually out. Or the iron, when red hot, may be buried in dry sawdust, and in that state allowed to anneal. One great advantage of annealing cast iron is, that if it is afterwards subjected to a partial heating, it is less liable to warp than it would otherwise be. The character of cast iron is not in any way altered by annealing, except it is rendered more malleable. But the greatest benefit arising from this process will be seen from the following rule, which is exceedingly useful to ironfounders, as by this means a great saving of expense may oftentimes be obtained in making new patterns.

To make a casting of precisely the same size of a broken casting without the original pattern.

Rule.—Put the pieces of the broken casting together and mould them, and cast from this mould.

When the casting is drawn from the sand, place it in a charcoal fire and anneal it, as above described; it will expand to the original size of the pattern, and there remain in that expanded state.

TENACITY OF METALS.

Mr. Guyton Morveau has carefully determined the weights which can be supported by wires of a uniform diameter of 0·787 of an English inch without fracture.

A wire of	iron supports	. . .	549·250 lbs.	avoirdupois.
,,	copper ,,	. . .	302·278	,,	,,
,,	platinum ,,	. . .	274·320	,,	,,
,,	silver ,,	. . .	187·137	,,	,,
,,	gold ,,	. . .	150·753	,,	,,
,,	zinc ,,	. . .	109·540	,,	,,
,,	tin ,,	. . .	34·630	,,	,,
,,	lead ,,	. . .	27·621	,,	,,
,,	steel ,,	(mean) .	898·023	,,	,,
,,	*do.* ,,	(maximum)	1098·500	,,	,,

TABLE ON THE SIZES AND STRENGTH OF SHORT-LINKED CRANE CHAINS AND ROPES, SHEWING THE DIMENSIONS AND WEIGHT OF EACH, AND THE PROOF OF THE CHAIN IN TONS.

Diameter of chains.	Weight per fathom.	Proof strain.	Circumference of rope.	Weight of rope per fathom.	Diameter of chains.	Weight per fathom.	Proof strain.	Circumference of rope.	Weight of rope per fathom.
inch.	lbs.	tons.	inch.	lbs.	inch.	lbs.	tons.	inch.	lbs.
$\frac{5}{16}$	6·	·75	2·$\frac{1}{2}$	1·5	$\frac{11}{16}$	28	6·5	7·	10·5
$\frac{3}{8}$	8·5	1·5	3·$\frac{1}{4}$	2·5	$\frac{3}{4}$	32	7·75	7·$\frac{1}{2}$	12·
$\frac{7}{16}$	11·	2·5	4·	3·75	$\frac{13}{16}$	36	9·25	8·$\frac{1}{2}$	15·
$\frac{1}{2}$	14·	3·5	4·$\frac{3}{4}$	5·	$\frac{7}{8}$	44	10·75	9·	17·5
$\frac{9}{16}$	18·	4·5	5·$\frac{1}{2}$	7·	$\frac{15}{16}$	50	12·5	9·$\frac{1}{2}$	19·5
$\frac{5}{8}$	24·	5·25	6·$\frac{1}{4}$	8·7	1·	56	14·	10·	22·

Chains for cranes should be made of short oval links, and should not exceed 1-inch diameter. The ropes of the sizes given are considered to be of equal strength with the chains, which being short-linked are made without studs, thus, (part across the centre is the *stud*). A crane chain will stretch, under a proof of fifteen tons, half an inch to a fathom.

METRIC SYSTEM.

WEIGHTS AND MEASURES.

CAPACITY.

Centilitre	=	$\frac{1}{100}$ of litre.
Décilitre	=	$\frac{1}{10}$ "
Litre	=	unit of capacity, a cubic décimètre.
Décalitre	=	10 litres.
Hectolitre	=	100 "
Kilolitre	=	1000 "

SOLIDITY.

Décistère	$= \frac{1}{10}$ of stère.
Stère	= unit of solidity, a cubic mètre (for firewood only).
Décastère	= 10 stères.

WEIGHT.

Milligramme	$= \frac{1}{1000}$ of gramme.
Centigramme	$= \frac{1}{100}$ „
Décigramme	$= \frac{1}{10}$ „
Gramme	= derivate unit of weight, the weight of a cubic centimètre of water.
Décagramme	= 10 grammes.
Hectogramme	= 100 „
Kilogramme	= 1000 grammes weight of a cubic décimètre of water.
Myriagramme	= 10 kilogrammes.
Metric quintal	= 100 „
Metric tonne	= 1000 „

SUPERFICIES.

Centiare	$= \frac{1}{100}$ are, or a square mètre.
Are	= unit of surface, a square decamètre.
Hectare	= 100 ares.

LENGTH.

Millimètre	$= \frac{1}{1000}$ of mètre.
Centimètre	$= \frac{1}{100}$ „
Decimètre	$= \frac{1}{10}$ „
Mètre	= fundamental unit of measure and weight.
Décamètre	= 10 mètres.
Hectomètre	= 100 „
Kilomètre	= 1000 „
Myriamètre	= 10,000 mètres.

The mètre having been found to be equivalent to 39·37079 *inches of the British* standard, we have the

Millimètre . . .	= 0·03937079 inch.
Centimètre . . .	= 0·03937079 „
Decimètre . . .	= { 3·937079 „ 0·328089916 foot.
Mètre . . .	= { 3 feet 3·37079 inches. 3·28089916 feet. 1·09363305 yard.

The decamètre being ten times the mètre, is equivalent, by last data, to 10·9363305 yards,

= 1 pole 5·4363305 yards.
= 1·9884237 pole.

The hectomètre being 100 mètres, is equal, by the data, to 109·36385 yards,

= 0·497105934 furlong.

The kilomètre being 1000 mètres, is equivalent, by last data, to 1093·63305 yards,

= 0·62138241792 mile.

The mètre being 39·37079 inches, it follows, by simple proportion, that the inch

= 0·02539954113188991, &c., mètre.
= 25·39954113188991, &c., millimètres.

By the last data the inch is equal to

0·02539954113188991, &c., mètre;

therefore, the foot = 0·304794493582, &c., mètre,

yard = 0.914383480748, &c., mètre.

Notice—The numbers of the Birmingham and other gauges correspond exactly or within ·001 of an inch with the numbers on the Whitworth wire gauge, except those marked thus*, which are within ·002, and those marked **, which are beyond ·002 of an *inch. Below* ·001, one thousand means ·002, two thousand parts *of an inch.*

WHITWORTH'S STANDARD WIRE GAUGE, WITH CORRESPONDING NUMBERS OF VARIOUS OTHER WIRE GAUGES.

Value of each number in decimals.	Number of Whitworth's standard wire gauge.	Numbers corresponding with Whitworth's gauge.				
		Birmingham wire gauge.	Lancashire wire gauge.	Old metal plate gauge.	Needle wire gauge.	Music wire gauge.
·001	1	..	..	..	..	..
·002	2	..	..	..	..	..
·003	3	..	..	..	..	..
·004	4	36	..	1	19	..
·005	5	35	..	2	18	..
·006	6	..	..	..	..	..
·007	7	34	..	..	17	..
·008	8	33	..	..	16	..
·009	9	32	..	..	15	..
·010	10	31	..	..	14	..
·011	11	..	..	..	..	..
·012	12	30	80	..	..	..
·013	13	29	79	..	12	..
·014	14	28	78	..	11	..
·015	15	..	77	..	..	..
·016	16	27	..	..	..	..
·017	17	..	76	..	..	..
·018	18	26	..	..	9	6
·019	19	..	75	..	8	7
·020	20	25	74	..	7	8
·022	22	24	72	10	6	10
·024	24	23*	71	..	..	12
·026	26	...	70	11	5	13
·028	28	22	68	..	..	14
·030	30	..	66	..	4	15
·032	32	21	64	12	..	16
·034	34	..	62	13	3	17
·036	36	20	61	..	..	18
·038	38	..	59	14	2*	19
·040	40	19*	56	15*	1	20*
·045	45	..	55	16	..	..
·050	50	18	54	17*	..	..
·055	55	..	52**	18	..	..
·060	60	17*	51	19	..	..
·065	65	16	49*	21*	..	..
·070	70	15*	47*	22*	..	..
·075	75	..	45	24*	..	..
·080	80	..	43**	..	..	..
·085	85	14*	42*	..	..	..
·090	90	..	41	25	..	..
·095	95	13	38	26**	..	..
·100	100	..	34	27**	..	..
·110	110	12	31*	28	..	..
·120	120	11	29	31*	..	..
·135	135	10	23**	34*	..	..
·150	150	9*	19	36*	..	..
·165	165	8	13**	..	..	..
·180	180	7	5**	..	..	..
·200	200	6**	2	..	..	..
·220	220	5	C*	..	..	..
·240	240	4*	G	..	..	..
·260	260	3	K	..	..	..
·280	280	2*	N*	..	..	..
·300	300	1	P*	..	..	..
·325	325	..	S*	..	..	..
·350	350	..	V*	..	..	..
·375	375	00**	X*	..	..	..
·400	400	..	..	..	..	..
·425	425	000**	..	..	..	..
·450	450	0000	..	..	..	..
·475	*475*	..	..	..	..	..
·500	*500*	..	..	..	..	..

USEFUL RECIPES.

The author introduces the following valuable receipts, which are often in request by mechanical men. They may be relied upon as being *bona fide*, and will be found to answer the various purposes required, provided the manipulation and quantities are correct:—

Glues.

Parchment Glue.—Parchment shavings one pound, water six quarts. Boil until dissolved, then strain, and evaporate slowly to the proper consistency. Use a water bath if wanted very light coloured.

Japanese Cement or Rice Glue.—Rice flour; water, sufficient quantity. Mix together cold, then bring the mixture to a boil, stirring it all the time. Observe to boil in a vessel that will not colour it.

Japanners' Gold Size.—Gum ammoniac one pound; boiled oil eight ounces; spirits of turpentine twelve ounces. Melt the gum, then add the oil, and lastly the spirits of turpentine.

Gold Size.—Yellow ochre one part; copal. varnish two parts; linseed oil three parts; turpentine four parts; boiled oil five parts; mix. The ochre must be in the state of the finest powder, and ground with a little of the oil before mixing.

Glue Liquid.—Glue, water, vinegar, each two parts. Dissolve in a water-bath, and add alcohol one part. An excellent cement.

Transparent Liquid Japan for Metal.—Copal varnish thirty-five parts; camphor one part; boiled oil two parts; mix.

Waterproof Glue.—Glue one part; skimmed milk eight parts. Melt and evaporate in a water-bath to the consistence of strong glue.

Papers.

Fire-proof Paper.—Take a solution of alum, and dip the paper into it, then throw it over a line to dry. This is suitable to all sorts of paper, whether plain or coloured, as well as textile fabrics. You must try a slip of the paper in the flame of a candle, and if not sufficiently prepared dip and try a second time.

Paper for Draughtsmen, &c.—Powdered tragacanth one part; water ten parts. Dissolve and strain through clean gauze, then lay it smoothly with a painter's brush on the paper, previously stretched on a board. This paper will take either oil or water colours.

Copying Paper.—Lay open your quire of paper (clean white, of large size), take the brush and cover it with the following varnish,

then hang it up on the line; take another sheet and repeat the operation, until you have finished your quantity. If not clear enough, give each sheet another coat when dry. Canada balsam, turpentine, equal parts; mix.

Liquid Gold for Vellum, &c.—Take gold-leaf and grind it with gum-water, then add a small quantity of bichloride of mercury, and bottle for use.

Liquid Silver for Vellum, &c.—Take silver-leaf and grind it with gum-water or glair of egg.

Waxed Paper.—Take cartridge or other paper, place it on a hot iron, and rub it with bees-wax; or make a solution of the wax in turpentine, and apply it with a brush. Useful for making water or air-proof pipes for chemical experiments, &c.

Tracing Paper.—Put oil four parts, turpentine five parts, mix and apply it to the paper, then rub it dry with wheat flour, and brush it over with ox-gall. This will bear writing on.

Lithographic Paper.—Give the paper three coats of thin size, one of starch, and one of solution of gamboge. Each to be applied with a sponge, and allowed to dry before the next is applied.

Hydrographic Paper.—This name has been applied to paper which may be written on with water. Use finely powdered nut galls and sulphate of iron heated till it becomes white. Pass the paper between rollers to allow the powdered galls to pass through it, or pass a heavy iron over the paper.

Chromotype Paper.—Soak the paper in a solution of bichromate of potash (in which solution a little sulphate of indigo is sometimes added to vary the colour), and dry it at a brisk fire. To fix the drawing, careful immersion in warm water is all that is required. This is not sufficiently sensitive for the camera.

Bronzing.

Bronzing Sculpture, Wood, &c.—Bronze of good quality acquires by oxidation a fine green tint, called *patina antiqua*. Corinthian brass receives, in this way, a beautiful clear green colour. This appearance is imitated by an artificial process called bronzing. A solution of sal-ammoniac and salt of sorrel in vinegar is used for bronzing metals. Any number of layers may be applied, and the shade becomes deeper in proportion to the number applied. For bronzing sculptures of wood, plaster-figures, &c., a composition of yellow ochre, Prussian blue, and lampblack, dissolved in glue-water, is applied.

Bronze.—Copper eighty-three parts, zinc eleven parts, tin four parts, lead two parts; mix. Copper fourteen parts, melt, and add *zinc six parts, tin four parts.*

Beautiful Red Bronze Powder.—Sulphate of copper 100 parts, carbonate of soda 60 parts. Apply heat until they unite in a mass, then cool, powder, and add copper filings 15 parts. Well mix, and keep them at a white heat for twenty minutes, then cool, powder, and wash and dry.

Bronzing Fluid for Guns.—Nitric acid sp. gr. 1·2, nitric ether, alcohol, muriate of iron, each one part; mix; then add sulphate of copper two parts, dissolved in water ten parts.

Enamels.

White Enamel.—Tin two parts, lead one part, calcine; then take of the above oxides one part, crystal two parts, manganese a small portion. Grind well together, fuse, and pour the mass into cold water; dry, grind again to powder, and fuse; repeat the process four or five times, observing great care to prevent any contamination from smoke, or iron, or copper.

Blue Enamel.—Fine paste (not metallic) ten parts, nitre three parts. Oxide of cobalt to colour.

Green Enamel.—Frit 1 pound, oxide of copper ½ ounce, red oxide of iron 12 grains.

Fluxes of Enamel Colours.—1*st.* Flint powder one part, calcined borax one part, flint glass three parts, red lead four parts. Keep them in a state of fusion, in a Hessian crucible, for three hours, then pour into cold water, dry, and powder. 2*nd.* Glass powder eleven parts, white arsenic one part, nitre one part; mix.

Yellow Enamel.—White oxide of antimony one part, white lead two parts, alum and sal-ammoniac each one part; mix in fine powder, and apply just sufficient heat to decompose the ammoniac.

Black Enamel.—Clay two parts, protoxide of iron one part; mix.

Marble Staining.

To stain Marble it is necessary to heat the marble hot, but not to such an extent as to injure it, the proper heat being that at which the colours nearly boil.

Blue.—Alkaline indigo dye, or turnsole with alkali.

Red.—Dragons blood in spirits of wine.

Yellow.—Gamboge in spirits of wine.

Gold Colour.—Sal-ammoniac, sulphate of zinc, and verdigris, equal parts.

Green.—Sap green in spirits, with potash.

Brown.—Tincture of logwood.

Crimson.—Alkanet root in turpentine.

The marble may be veined according to taste. To stain marble *well* is a tedious and difficult operation.

To stain White Marble—Apply with a brush a strong alcohol tincture, made from the root *alkanet.*

To extract Oil from Stone or Marble.—Soft soap one part, Fuller's earth two parts, potash one part, boiling water to mix. Lay it on the spots of grease, and let it remain for a few hours.

Dyeing.

Japan Grounds—Red.—Vermillion makes a fine scarlet, but its appearance in Japanned work is much improved by glazing it with a thin coat of lake, or even rose pink.

King's-yellow, or turpeth mineral, and Dutch pink, all form very bright yellows, and the latter is very cheap.

Green.—Distilled verdigris laid on a ground of leaf gold produces the brightest of all greens; other greens may be formed by mixing king's-yellow and bright Prussian blue, or turpeth mineral and Prussian blue, or Dutch pink and verdigris.

White.—White grounds are obtained with greater difficulty than any other. One of the best is prepared by grinding up flock-white or zinc-white with one-sixth of its weight of starch, and drying it; it is then tempered like the other colours, using the mastic varnish for common uses, and that of the best copal for the finest. Particular care should be taken that the copal for this use be made of the clearest and whitest pieces. Seedlac may be used as the uppermost coat, where a very delicate white is not required, taking care to use such as is least coloured.

Black.—Ivory-black, or lampblack; but if the lampblack be used it should previously be calcined in a closed crucible. Black grounds may be formed on metals by drying linseed oil only when mixed with a little lampblack. The work is then exposed in a stove to heat, which will render the oil black. The heat should be low at first, and increased very gradually, or it will blister. This kind of Japan requires no polishing. It is extensively used for defending iron articles from rust.

Polishes.

To brass Plates of Copper.—The plates being previously heated expose them to the fumes of zinc.

To brass Vessels of Copper.—Argol one part; amalgam of zinc one part; muriatic acid two parts; water to fill the vessel; boil.

French Polish.—Alcohol 260 parts; copal varnish thirteen *parts; sandarach* (powdered) one part; mastic (powdered) one

part; shell-lac (powdered) twenty-four parts. Mix and digest in a moderate heat in a strong close vessel.

Gilding.

To gild Copper, Brass, &c., (patent).—Fine gold five parts; nitric acid (specific gravity, 1·45) twenty-one parts; hydrochloric acid (specific gravity, 1·15) seventeen parts; pure water fourteen parts. Digest with heat in a glass vessel until all the gold is dissolved, and till red or yellow fumes cease to rise. Decant the clear liquid into some convenient vessel, and add water, 500 to 600 parts. Boil for two hours, let it stand to settle, and pour off the clear into a suitable vessel. For use, heat the liquid and suspend the articles (previously well cleaned) by means of a hair of fine wire, until sufficiently coated with gold, then well wash them in pure water.

To give Iron the Colour of Copper.—Take 1 ounce of copper-plates, cleansed in the fire; 3 ounces of aquafortis; dissolve the copper, and when it is cold use it by washing your iron with it by the help of a feather; it is presently cleaned and smooth, and will be of a copper colour; by much using or rubbing it will wear off, but may be renewed by the same process.

A way of Gilding with Gold upon Silver.—Beat a ducat thin, and dissolve it in 2 ounces of aquafortis, dip clean rags in it, and let them dry; burn the rags, and with the tinder thereof rub the silver with a little saliva. Be sure first that the silver be free from grease.

To make Copper into a Metal like Gold.—Distilled verdigris 4 ounce; Tutiæ Alexandrinæ præparatæ 2 ounces; saltpetre 1 ounce; borax half an ounce. Mix all together with oil till they be as thick as pap, then melt it in a crucible and pour it into a fire-shovel, first well warmed.

To gild or silver Writing.—Let there be a little gum and lump-sugar in the ink you write with; when dry breathe on it and apply the leaf.

To gild Steel.—Apply an ethereal solution of gold. This is equally adapted to lettering, as wholly covering the object. It may be applied with a pen or otherwise.

Factitious Stones.

AMETHYST.

Take strass, 5000 parts; oxide of manganese, thirty-seven parts *oxide of cobalt*, twenty-five parts; purple of cassius, one part. *Fuse for* twenty-six hours and cool slowly.

EMERALD.

Oxide of chrome, one part; green oxide of copper, twenty parts; strass, 2300 parts. Fuse with care for twenty-six hours, then cool slowly.

ARTIFICIAL CORAL.

Yellow resin, four parts; vermillion, one part; melt. This gives a very pretty effect to glass twigs, cinders, stones, &c., dipped into it. It is also useful for a cement for ladies' fancy-work, such as grottoes, &c.

OPAL.

Strass, 500 parts; horn silver, ten parts; calcined magnetic ore, two parts; chalk marl, twenty-five parts. Mix in fine powder and fuse with great care.

ORIENTAL RUBY.

Strass, 7000 parts; precipitate of cassius and nitric peroxide of iron, each 165 parts; golden sulphuret of antimony, 160 parts; manganese, calcined with nitre, 150 parts; rock crystal, 1000 parts. Mix in fine powder and carefully melt.

SAPPHIRE.

Paste of strass, 2300 parts; oxide of cobalt, thirty-four parts. Fuse carefully for thirty hours.

TOPAZ.

Strass, 1000 parts; glass of antimony, forty-two parts; purple of cassius, one part. Fuse for twenty-four hours and cool slowly.

DIAMOND.

Manganese, one part; rock crystal, 2800 parts; borax, 1900 parts; white lead, 5700 parts. Mix in fine powder, then fuse in a clean crucible, pour it into water, dry, powder, and repeat the process two or three times.

PART VII.—THE STEAM-ENGINE.

GENERAL RULES.

Steam Cylinder, Thickness.—(*Vertical*)—Multiply its diameter by the extreme pressure of steam in pounds per square inch that it may be subjected to, and divide the product by 2400; the result will give the thickness in inches. (*Horizontal*)—Divide as above, by 2000. (*Inclined*)—Divide as above, in a ratio inversely as the sine of the angle of inclination.

Shafts, Gudgeons, &c.—To resist torsion.

EXAMPLE.—The diameter of the journal of a shaft, to resist torsional stress, is ascertained to be 17 inches, and the diameter of its body in the centre, to resist lateral stress, has also been ascertained to be 14·422 inches, what should be the diameter of the body?

$\sqrt[3]{(17^3 + 14{\cdot}422^3)}$, and $\sqrt[3]{7913} = 19{\cdot}927$ inches.

N.B.—The strength of a cylindrical shaft compared to a square one, the diameter of the one being equal to the side of the other, is as 1 to 1·2, and of a square shaft to a cylindrical as 1 to ·85.

Journals of Shafts.—Their length should be from 1·15 to 1·25 times their diameter, and in *main centres* they should be 1·5 times.

The *bearings* of driving and propeller shafts are not here considered as journals.

Cross Heads, Wrought Iron (*Cylinder*).—Its section in the centre should be determined by the following formulæ:—

$$\frac{a \times p \times l}{700} = s, \text{ and } \sqrt{\frac{s}{b}} = d \text{ or } \frac{s}{d^2} = b;$$

a representing area of cylinder in inches, p the extreme pressure in pounds per square inch that it may be subjected to, l the length of the cross head between the centres of its journals in feet, and s the product of the square of the depth d, and the breadth b, of the section.

(*Air-pump*). $\frac{a \times b}{18} = s$, and as above for d and b.

If the section of either of these cross heads is cylindrical, for s put $\sqrt[3]{s \times 1{\cdot}7}$.

Diameter of boss twice, and diameter of end journals the same as that of the piston rod. Section at ends ·5 that of their centre.

Steam Pipe.—Its area should exceed that of the steam valve.

Connecting Rod.—Its length should be 2·25 times the stroke of the piston when it is at all practicable to afford the space. When, however, it is imperative to reduce this proportion, it may be a little less than twice the stroke.

N.B.—The comparative friction of long and short connecting rods is, for once the length of stroke of piston 12 per cent. additional; twice, 3 per cent.; and thrice, 1·33.

Neck.—Its diameter should be from 1 to 1·1 that of the piston rod.

Centre of the Body (*Horizontal*).—Its diameter is ascertained in e following manner:—

Multiply the length of the body (between the necks) in feet by e area of a neck in inches, divide the product by $\frac{3}{4}$ the stroke of e piston or the throw of the rod in feet, and the quotient is the ea of the centre in square inches, from which the diameter may determined.

(*Vertical*).—The area deduced by this rule may be somewhat s.

Connecting Links.—Their length should be ·5 of that of the oke, or of the throw of their attachments.

N.B.—Where a pair of connecting rods is used, as in some scriptions of engines, with a pair of connecting links, their cks should have an area of ·7 to ·75 of that of the attached rod. hen a second set of connecting rods or links is used, as with me air-pump connections, &c., their necks should have an area a ratio inversely as their throw to that of the first set.

Straps of Connecting Rods, Links, &c.—The area of the *strap* at least section should be ·65 that of the neck of the attached rod; e *key* should be in thickness ·3 the diameter of the neck; the idth of *gib* and *key* combined should be 1·25 times; and the *slot* ould be 1·35 times that of the same diameter. The *draft* of eys should be ·5 to ·6 of an inch per foot; distance of slot from nd of rod ·5 diameter of pin.

Pins, for Cranks, Beams, &c.—Their area at their journals hould be 1·6 to 1·75 times; and for *air pump* connections their rea should be 1·5 times that of the attached rod; their length 1·3 o 1·5 times their diameter.

Cranks (*Wrought Iron*).—The *hub*, compared with the neck of the haft, should be 1·75 the diameter and once the depth. The *eye*, compared with the pin, should be twice its diameter, and 1·5 the

depth. The *web*, at the periphery of the hub, should be, in width, ·7 the width, and in depth ·5 the depth of the hub; and at the periphery of the eye it should be, in width, ·8 the width, and in depth ·6 the depth of the eye.

(*Cast Iron*).—The diameters of the *hub* and *eye* should be respectively twice the diameter of the neck of the shaft, and 2·25 times that of the crank pin.

The *radii* for the fillets of the sides of the web should be one-half of the width of the web at the end of which the fillet is designed; for the fillets at the back of the web, they should be one-half of that of the sides of their respective ends.

Beams, Open or Trussed.—Their length from centres should be 1·8 to 2 the stroke of the piston, and their depth ·5 their length. If strapped, the *strap* at its smallest dimensions should have at least ·9 the area of the piston rod, its depth being equal to ·5 its breadth. The *end centre journals* should have each once, and the *main centre journals* should have each 2·5 times the area of the piston or drawing rod.

N.B.—This proportion for the strap is as above, when the depth of the beam is half of its length; consequently, when its depth is less, the area of the strap must be increased; and when the depth of the strap is greater or less than half its width, its area is determined by the product of its bd^2, being the same as if its depth was half its width.

(*Cast Iron*)—*Area of Section at Centre.*—Multiply the extreme pressure upon the piston in pounds by half the length of the beam, and divide the product by 500 times the depth of the centre in inches, which should be ·5 to ·75 the diameter of the cylinder, and when of uniform thickness of not less than ·1 of their depth.

Plumber Blocks (*Shaft*).—Bottom ·4, and binder ·45, the diameter of the shaft journal.

Holding-down Bolts.—If two are used ·3 to ·33, and if four, ·22 to ·25, the diameter of the shaft.

Cocks.—The angles of the sides of their plugs should be from 7° to 8° from the plane of it.

Pumps.—The velocity of water in pump openings should not exceed 500 feet per minute.

Fly Wheels and Governors.—The operation of the governor or conical pendulum depends upon the principles of central forces. If a governor makes six revolutions in a second when the ring is 16 inches from the fixed point or top, the distance of the ring will be 5·76 inches when the speed is increased to ten revolutions in the same time.

For 10 : 6 :: $\sqrt{16}$: 2·4, which squared = 5·76 inches, the *distance of the* ring from the top. Or, $6^2 : 10^2 :: 5{\cdot}76 : 16$ inches.

A *governor* performs in one minute half as many revolutions as a pendulum vibrates, the length of which is the perpendicular distance between the plane in which the balls (ball governor) move and the fixed point or centre of suspension.

Smoke Pipes and Chimneys.—Their area at their base should exceed that of the extremity of the flue or flues with which they are connected.

The intensity of their draught is as the square root of their height.

The relative volumes of their draught is determined by the formula:

$\sqrt{h} \cdot 1\ a$ = volume in square feet; h representing the height of the pipe or chimney in feet, and a its area in square feet.

When wood is consumed, their area should be 1·6 times that for coal.

BOILERS.

Natural Draught.

Boilers (*Land*).—Should be set at an inclination of ·5 inch in 10 feet.

Grates (*Coal*).—They should have a superficial area of 1 square foot for every 15 lbs. of coal required to be consumed per hour, at a rapid rate of combustion, and they should be set at an inclination toward the bridge-wall of 1 inch in every foot of length.

When, however, the rate of combustion is not high, in consequence of the low velocity of the draught of the furnace, or the fuel being insufficient, this proportion must be increased to 1 square foot for every 12 lbs. of fuel.

Ash-pit.—Its transverse area for a combustion of 15 lbs. of coal per hour, should be ·25 the area of the grate surface for bituminous coal and ·33 for anthracite.

The velocity of the current of air entering an ash-pit may be estimated at 12 feet per second.

Furnace or Chamber (*Coal*).—Its volume should be from 2·75 to 3 cubic feet for every square foot of grate surface.

Combustion is most complete with firings or charges at intervals of from fifteen to twenty minutes. The volume of air and smoke for each cubic foot of water converted into steam is from *coal 1780 to 1950 cubic* feet, and from wood 3900.

Bridge-wall (*Flue Boilers*).—The cross section of the flues or tubes should have an area of 1·7 to 2 square inches for each pound of coal consumed per hour, or from 22·5 to 26 square inches for each square foot of grate, for a combustion of 13 lbs. of coal per hour; the difference in the area depending upon the character of the conformation of the section of, and the length of the passage of the gases—the area being inversely as the diameter and directly as the length of the flues, tubes, or spaces between them. Thus, in *horizontal tubular* boilers, the area should be increased to 27·5 and 31 square inches; in *vertical tubular* to 32·5 and 36 square inches; and when a blast is used the area may be increased to 15·5 and 20·5 square inches.

The temperature of a furnace is about 1000°, and the volume of air required for the combustion of 1 lb. of bituminous coal, together with the products of combustion, is 154·81 cubic feet, which, when exposed to the above temperature, makes the volume of heated air at the bridge-wall from 450 to 470 cubic feet for each pound of coal consumed upon the grates.

Hence, at a velocity of the draught of about 36 feet per second, the area over a bridge-wall required to admit of this volume being passed off in an hour, would be ·5 of a square inch, but in practice it should be 2 square inches.

The temperature of the heated air at the end of the flues should be about 500°, and their area, and that of the base of the chimney, should be ·75 of that over the bridge-wall, or 1·5 square inches for each 13 lbs. of coal consumed per hour.

When the area of the flues is determined upon, and the area over the bridge-wall is required, it should be taken at from ·7 to ·8 the area of the lower flues for a natural draught, and from ·5 to ·6 for a blast.

Flues.—Their area should increase with their length, but not in proportion with the reduction of the temperature of the heated air, their area at their termination being from ·7 to ·8 that of their *calorimeter**, which should have an area of 1·33 square inches for every pound of coal consumed per hour. With tubes of small diameter compared to their length, this proportion may be reduced to 1 inch.

Evaporation.—1 square foot of grate surface, at a combustion of 13 lbs. coal per hour, will evaporate 2 cub. ft. of salt water per hour.

A square foot of heating surface, at the above combustion of 13 lbs., will evaporate from 4·33 to 5·33 lbs. of salt water per hour.

Note.—The boilers of the S. S. "Arctic" (*vertical tubular*) have a surface of 33¼ to 1 of grate, consuming 13 lbs. of coal per square

* The *area* immediately behind the bridge-wall.

foot of grate per hour; the evaporation is 8·56 lbs. of salt water per pound of coal, including that lost by blowing out saturated water.

Water Surface.—At low evaporations 3 square feet are required for each square foot of grate surface, and at high evaporations 4 to 5 square feet.

HEATING SURFACES.

Heating Surfaces (*Sea Water*).—The grate and heating surfaces should be increased about ·07 over that for fresh water.

RELATIVE VALUE OF HEATING SURFACES.

Horizontal surface above the flame, . . .	= 1
Vertical „	= ·5
Horizontal „ beneath the flame, . .	= ·1
Tubes and flues, ...	= ·56

N.B.—A scale $\frac{1}{16}$ of an inch in thickness will effect a loss of 14·7 per cent. of fuel.

One square foot of *fire* surface is computed to be as effective as three of *heating* surface.

When the combustion in a furnace is complete, the tubes may be shorter than when it is incomplete.

Tubes should always be set in vertical rows, and the spaces between them should be increased with their number.

To compute the heating and grate surface required for a given evaporation, or volume of cylinder and revolutions.

Operation.—Reduce the evaporation to the required volume of cylinder, number of revolutions of engine, pressure of steam, and point of cutting off; then reduce these results to the range of consumption of fuel per square foot of grate, pressure of steam, and number of revolutions given for the several following cases, and multiply them by the units given for the surfaces required.

Internal Furnaces (*Boilers*).—Coal, 13 lbs. per hour per square foot of grate (natural draught). Pressure of steam, 20 lbs. (mercurial gauge), and 20 revolutions of the engine per minute.

Coal, 30 lbs. (mercurial gauge), 20 revolutions per minute. Steam pressure, 30 lbs.

External Furnace and Internal Flues (*Cylindrical Flue*).—Coal, 20 lbs. *per hour* per square foot of grate (natural draught).

Pressure of steam, 100 lbs. (mercurial gauge), and 20 revolutions of the engine per minute.

Coal, 20 lbs.; pressure, 100; revolutions, 40 per minute.

The above examples may be easily computed by the rule laid down under the heading "Operation."

EXAMPLE.—There is required an evaporation of 492·24 cubic feet of salt water per hour, under a pressure of steam of 17·3 lbs. per square inch, stroke of engine 10 feet, cutting off at $\frac{1}{2}$ stroke, revolutions per minute 15, and consumption of coal 13 lbs. per square foot of grate per hour, in a marine boiler having internal furnaces and vertical tubes.

Volume of steam at this pressure compared with water, 833.

$492{\cdot}24 \times 833 \div 60 = 6833{\cdot}93$ *cubic feet* of cylinder per minute.

$6833{\cdot}93 \div \overline{15 \times 2} = 227{\cdot}79$ cubic feet of cylinder at $\frac{1}{2}$ stroke.

Then, $\frac{227{\cdot}79 \times 17{\cdot}3}{20} = 197{\cdot}04$ cubic feet at 17·3 lbs. *pressure*, and $\frac{197{\cdot}04 \times 15}{20} = 147{\cdot}78$, which × 66, the unit for *heating* surface for a *vertical tubular* boiler at 20 *lbs. pressure*, and 20 *revolutions* = 9753·48 *square feet*.

And $147{\cdot}78 \times 2 =$ the *unit for grate* under like condition = 295·56 *square feet*.

Note.—The steamer "Baltic" has developed all the elements here given, and the surfaces of her boilers and grates (for one engine) were 9742 and 293·9 *square feet*.

To compute the consumption of fuel in the furnace of a boiler.

The dimensions of the cylinder, the pressure of the steam, the point of cutting off, the revolutions, and the evaporation of the boilers per pound of fuel per minute being given.

Rule.—Ascertain the volume of water expended in steam, and multiply it by the weight of a cubic foot of the water used; divide the product by the evaporating power of the fuel in the boiler under computation in pounds of water, and add thereto the loss per cent. by blowing off.

BOILER PLATES, BOLTS, AND JOINTS.

The tensile strength of iron plates and bolts ranges from 42,000 *to 62,500* lbs., being increased when subjected to a moderate *temperature*.

The mean tensile strength of copper plates and bolts is 33,000 lbs., being reduced when subjected to a temperature exceeding 120°, at 212° being 32,000 lbs., and at 550° but 25,000 lbs.

Bursting and Collapsing Pressure.

For use in salt water, computation for iron plates or bolts, without reference to the riveting, should be based upon a strength of two-fifths that of the ultimate strength of the metal; and for use in fresh water upon one-half that of its ultimate strength. With copper, one-half is a safe reduction for all purposes.

The resistance to collapse of a flue or tube is much less than the resistance to bursting; the ratio cannot be determined, as the resistance of a flue decreases with its length, or that of its courses.

With an ordinary cylindical boiler 4 feet in diameter, single riveted, 20 feet in length, with flues $15\frac{1}{2}$ inches in diameter; shell, $\frac{5}{16}$ thick; flues, $\frac{1}{4}$ inch, the relative strengths are: bursting 350 lbs., collapsing, 152 lbs.

The following units are based upon a tensile strength of iron of 52,500 lbs., and for copper of 32,000 lbs.

To compute the thickness, maximum working pressure, and diameter of an iron boiler or flue (for service in salt water).

Thickness.—Rule—Multiply the diameter in feet by the working pressure in pounds; divide the product by 1260 for square riveting, 1170 for staggered, and 900 for single, and the quotient will give the thickness in decimals of an inch.

Working Pressure.—Rule—Multiply the thickness by 1260, 1170, or 900, as before given; divide the product by the diameter in feet, and the quotient will give the pressure in pounds.

Diameter.—Rule—Multiply the thickness by 1260, 1170, or 900, as before given; divide the product by the working pressure, and the quotient will give the diameter in feet.

Example.—The diameter of a single-riveted iron boiler is 4 feet, and the thickness of the plates $\frac{5}{16}$, what will be the maximum working pressure?

$$\text{Here } \tfrac{5}{16} = \cdot 3125 \text{ and } \frac{\cdot 3125 \times 900}{4} = 70 \cdot 3 \text{ lbs.}$$

N.B.—For service in fresh water the preceding units are *increased one-fourth*—viz.: 1575, 1460, and 1125.

To compute the diameter of stay bolts.

Rule.—Multiply the distance between their centre in inches by the square root of the quotient of the maximum working pressure; divide by 5530 for salt water, and 6900 for fresh for iron bolts, and by 5000 for copper bolts, and the quotient will give the distance apart in inches.

The strength of iron stay bolts (for salt water) should be computed at $\frac{1}{7}$ of the ultimate strength of the metal.

For fresh water the strength of iron stay bolts should be computed at $\frac{1}{6}$ of the ultimate strength of the metal.

The strength of copper bolts may be taken at $\frac{1}{5}$ the strength of the metal, either for salt or fresh water.

EXAMPLE.—The maximum working pressure of an iron boiler, for use in salt water, is 70 lbs., and the distance apart of the bolts is 8 inches; what should be their diameter?

$$8 \times \sqrt{\frac{70}{5530}} = 8 \times \sqrt{\cdot 01266} = 8 \times \cdot 1125 = \cdot 9 \text{ inches.}$$

To compute the distance apart of stay bolts.

Rule.—Multiply the square root of the quotient of 5530 for salt water, and 6900 for fresh water, for iron bolts, and by 5000 for copper bolts; divide by the maximum working pressure by the diameter of the bolts, and the product will give the distance in inches.

EXAMPLE.—The maximum working pressure of an iron boiler for use in salt water is 70 lbs., and the diameter of the stay bolts is 9 inches; what should be their distance apart?

$$\sqrt{\frac{5530}{70}} \times \cdot 9 = \sqrt{79} \times \cdot 9 = 8 \text{ inches.}$$

To compute the thickness of flat surfaces in a boiler.

Rule.—Multiply the maximum working-pressure by the square of the distance or the area of the surface between the centres of the stays in inches; divide the product by 45,700, and the square root of the quotient will give the thickness in inches.

EXAMPLE.—Take the elements of the preceding case.

$$\text{Then } \sqrt{\frac{70 \times 8^2}{45700}} = \sqrt{\cdot 098} = \cdot 313 \text{ inches.}$$

N.B.—For fresh water take 57000, and for copper 41300.

Stay Bolts.—Iron stay bolts $\frac{3}{4}$ inches in diameter, and screwed into a copper plate $\frac{3}{8}$ thick, drew at a strain of 18260 lbs.

A like stay bolt, screwed and riveted into an iron plate, drew at a strain of 28670 lbs.

A like stay bolt of copper, screwed and riveted into a copper plate, drew at a strain of 16265 lbs. Hence, stay bolts when screwed and riveted are $\frac{1}{3}$ stronger than when screwed alone.

TABLE OF FORMS AND PROPORTIONS OF RIVETED JOINTS.

Thickness of plate.	Diameter of rivets.	Multiplier.	Length of rivets.	Multiplier.	Centre to centre of rivets.	Multiplier.	Lap in single joints.	Multipliers.	
								Single joints.	Double joints.
Inches.	Inches.		Inches.		Inches.		Inches.		
$\frac{3}{16}$	$\frac{3}{8}$	2	$\frac{7}{8}$	4·5	$1\frac{1}{4}$	6·5	$1\frac{1}{4}$	6·8	11·1
$\frac{1}{4}$	$\frac{1}{2}$	2	$1\frac{1}{16}$	4·5	$1\frac{1}{2}$	6	$1\frac{1}{2}$	6	10
$\frac{5}{16}$	$\frac{5}{8}$	2	$1\frac{5}{16}$	4·5	$1\frac{5}{8}$	5·2	$1\frac{7}{8}$	6	10
$\frac{3}{8}$	$\frac{3}{4}$	2	$1\frac{5}{8}$	4·5	$1\frac{3}{4}$	4·7	2	5·3	8·8
$\frac{7}{16}$	$\frac{13}{16}$	1·5	$2\frac{1}{4}$	4·5	2	4	$2\frac{1}{4}$	4·5	7·5
$\frac{1}{2}$	$\frac{15}{16}$	1·5	$2\frac{3}{4}$	4·5	$2\frac{1}{2}$	4	$2\frac{1}{4}$	4·4	7·3
$\frac{9}{16}$	$1\frac{1}{8}$	1·5	$3\frac{3}{8}$	4·5	3	4	$3\frac{1}{4}$	4·3	7·2

The length of a rivet, as of a bolt, is measured from inside of its head.

The multipliers are for computing the diameter, length, and distance between centres of the rivets; also for the laps for single and double joints, by multiplying the thickness of the plate by the multiplier for the element required.

In riveted joints exposed to a tensile strain, the area of the rivets should be equal to the areas of the section of the plates through the line of the rivets, running a little in excess up to $\frac{9}{16}$ inches, and somewhat less beyond that diameter of rivet.

HORSE POWER.

As this is the universal term used to express the capability of first movers of magnitude, it is essential that the estimate of it should be uniform.

Its estimate is the elevation of 33000 pounds avoirdupois 1 foot in height in one minute, and it is designated as being *nominal, indicated,* or *actual.*

The first designation being adopted and referred to by manufacturers of steam-engines in order to express the capacity of an engine, the elements thereof being pressure of steam and speed of piston; the second, to designate the full capacity of an engine, as developed in operation, without any deduction for friction; and the last referring to its actual power as developed by its operation, involving the elements of the mean pressure upon the piston, its velocity, and a just deduction for the friction of the operation of the machine.

In reviewing the various modes for the computation, as submitted by engineers and manufacturers, there is no proper formula that presents the *essential* element of being in *conformity* with any other; and as conformity in a rule for this purpose, if based upon an assimilation to the capacity of an engine, is all that is requisite, it would have been preferable to have adopted an existing formula to the introduction of a new one, had it been practical to have done so. It occurs, further, that there is not only a want of conformity in the various rules essayed by authors, but they have neither reached the case of both condensing and non-condensing engines, nor have they properly approached to the actual power of an engine; and as the practice of operating engines since the adoption of existing formulæ has materially altered both in an increase of pressure and velocity of piston, the following rules are submitted :—

Nominal horses' power (*condensing engine*).

$\frac{d^2v}{3000}$ = horses' power: d representing diameter of cylinder in inches, and v the velocity of the piston in feet per minute.

This is similar to the rule of the Admiralty, substituting 3000 for 6000, and it is based upon a uniform steam pressure of 10 lbs. per square inch (steam gauge, or *above* the pressure of the atmosphere), cut off at one-half the stroke, deducting one-fifth for friction and losses, with a mean velocity of piston of 250 feet per minute for an engine of long stroke, and of 200 feet for one of short stroke.

The rule of the Admiralty is based upon a uniform and effective pressure of 7 lbs. per square inch at full stroke, and a mean velocity of piston of 205 feet per minute—viz., 170 feet for a stroke of 2·5 feet, and 240 feet for a stroke of 8 feet.

Non-condensing engine. — $\frac{d^2v}{1000}$ = horses' power. This is based upon a uniform steam pressure of 60 lbs. per square inch (steam gauge), cut off at one-half the stroke, deducting one-sixth for friction and losses, with a mean velocity of piston of 250 feet per second.

Indicated horse power.—This is the gross power exerted by an engine, without any deduction for friction, the mean pressure upon the piston being determined by an indicator, or by a computation based upon the actual initial pressure in the cylinder.

Actual or effective power (condensing engine).— $\frac{A \times \overline{P^* - f^\dagger} \times 2sr}{33000}$ = horses' power; A representing area of cylinder in square inches, P mean effective pressure upon cylinder piston in pounds per square inch, inclusive of the atmosphere, *f* the friction of the engine in all its parts, added to the friction of the load in pounds per square inch, *s* stroke of piston in feet, and *r* number of revolutions per minute. The power required to work the air-pump of an engine varies from ·7 to ·9 lbs. per square inch upon the cylinder piston.

Illustration.—The diameter of cylinder of a marine steam-engine is 60 inches, the stroke of its piston 10 feet, its revolutions 15 per minute, and the pressure of the steam gauge, cut off at one-fourth the stroke, is 20 lbs. per square inch.

A = 2827·4 square inches, P 20·855‡ lbs., $f = 1{\cdot}5 + (20{\cdot}855 - 1{\cdot}5) \times 05 = 2{\cdot}467$ lbs. Then $\frac{2828{\cdot}4 \times (20{\cdot}855 - 2{\cdot}467) \times 2 \times 10 \times 15}{33000} = 533{\cdot}24$ horses' power. From which is to be deducted in marine engines the power necessary to discharge the water of condensation at the level of the load-line, which is determined by the pressure due to the elevation of the water, the area of the air-pump piston, and the velocity of its discharge in feet per second.

With a view of illustrating the relative capacities of *screw propellers* and *side wheels*, and to exemplify the indicated horse power of both classes of steamers, the following particulars are submitted :—

H.M.S. "Arrogant" (screw propeller) and side wheel steamer "Pacific" had very similar dimensions, like draughts of water and area of immersed amidship sections. But the "Pacific" being 75 feet longer, consequently had easier lines.

Result of Performance.

"Arrogant," indicated horses' power 729. Speed (knots) 8·35.
"Pacific," indicated horses' power 1964. Mean speed in knots per hour 12.

* Add 14·7 lbs., for the atmospheric pressure, to that indicated in the steam gauge or safety valve.

† Estimate this value in engines of magnitude at 1·5 to 2 lbs. per square inch, *for the friction of the* engines in all parts.

‡ *Clearance of piston* at each end of cylinder is included in this estimate.

Then, $8{\cdot}35^3 = 582$ and $12^3 = 1728$. Therefore

$$1964 : 1728 :: 729 : 663 \text{ and } \frac{582}{663} = {\cdot}877.$$

Hence, the application of power in the "Pacific" exceeded that in the "Arrogant" as 1 to ·877.

This vessel in 1858 steamed 1665 miles in six days and one hour.

The "Adriatic," with wheels of a diameter of 40 feet, arms 32, blades 32, and a dip at load line of 8·25 feet, ran, in 1860, 365 knots in 24 hours, and at Stokes Bay averaged a speed of 15·9 knots to the measured hour.

The "Ville de Paris," a powerful screw steamer, built eight years after the "Adriatic's" performance, averaged 340 miles per day on a trip from Sandy Hook, New York, to Brest, France.

The capacity in both cases, as regards indicated horse power, being in preponderance in the screw-propellers.

PART VIII.—THE LOCOMOTIVE ENGINE.

PROPORTION of parts based upon diameter of cylinder:

Crank Shaft	Diameter	·4
Connecting Rod, end	„	·16
Connecting Rod, middle	„	·21
Piston Rod	„	·15
Depth of Piston		·28
Blast Pipe	Diameter	·3
Area Steam Port	$d^2 \times$	·08
Area Exhaust Port	$d^2 \times$	·28

$\cdot 0022 \times \frac{H}{g} = w$; $21 \cdot 2 \sqrt{wg} = H$; $\cdot 0022 \frac{H^2}{w} = g$; H and g representing areas of heating and grate surface in square feet, and w volume evaporated in cubic feet per hour.

Heating Surface—70 to 85 times that of the grate surface.

Grate Surface.—Area ·011 of heating surface when coke is used, and ·016 when bituminous coal is used.

Area of tube surface 10·5 times that of the furnace surface.

Steam Room—6 to 7 cubic feet per square foot of grate surface.

The evaporation of water in a locomotive engine (boiler) is from 8·5 to 9·5 lbs. water, from 212°, per lb. of *coke* consumed.

MEMORANDA.

1192 square feet of heating surface and 74 feet of grate in a locomotive boiler (tubular direct), gave identical results with a plane cylindrical boiler having 1184 feet of heating surface and 96 feet of grate.

The mean results of an experiment with a passenger train was 18 lbs. coke expended per mile run, and 26·3 lbs. per mile at a speed of 24·5 miles.

There is an increase of effect with large *drums* over small. ·76 *cubic feet of water* evaporated per hour will produce 1 horse power.

In the Sæmmering locomotive a power of 4·2 horses has been attained per ton of engine and tender; and 1 lb. of wood (fuel) has raised 13,600 lbs. 1 foot in height per minute = 14·6 lbs. wood per horse-power; and in a freight locomotive (No. 227) used on the North-Western Railway, a power of 3·55 horses has been attained per ton of engine and tender.

SPEED OF TRAINS.

60 ÷ minutes in running 1 mile = miles per hour.
3600 ÷ seconds in running 1 mile = miles per hour.

Railway Train.—At a speed of 33 miles per hour a distance of 57 yards is required within which a train can be arrested; and at a speed of 63 miles per hour a distance of 273 yards is required.

Brakes.—The resistance of brakes, as determined by experiments, is about 129 lbs. per ton of train.

TABLE OF THE VELOCITY OF PISTONS, &c.

Stroke of the Pistons.	Diameter of Driving Wheels.								
Inches.	4 ft. 0 in.	4 ft. 6 in.	5 ft. 0 in.	5 ft. 6 in.	6 ft. 0 in.	6 ft. 6 in.	7 ft. 0 in.	7 ft. 6 in.	8 ft. 0 in.
20	0·2652	0·2393	0·2122	0·1929	0·1768	0·1632	0·1516	0·1414	0·1326
19	0·2519	0·2273	0·2016	0·1832	0·1679	0·1550	0·1440	0·1343	0·1259
18	0·2386	0·2153	0·1910	0·1736	0·1591	0·1468	0·1364	0·1272	0·1194
17	0·2254	0·2034	0·1803	0·1640	0·1503	0·1387	0·1288	0·1202	0·1127
16	0·2221	0·1914	0·1697	0·1543	0·1415	0·1305	0·1213	0·1131	0·1061
15	0·1989	0·1795	0·1591	0·1447	0·1326	0·1224	0·1137	0·1060	0·0994
14	0·1856	0·1675	0·1485	0·1350	0·1237	0·1141	0·1061	0·0990	0·0928
13	0·1724	0·1555	0·1379	0·1254	0·1149	0·1061	0·0985	0·0919	0·0862
12	0·1591	0·1436	0·1273	0·1157	1·1061	0·0979	0·0909	0·0848	0·0796

APPLICATION OF THE PRECEDING TABLE FOR FINDING THE TRACTIVE POWER OF LOCOMOTIVE ENGINES.

Multiply the sum of the areas of the two pistons by the effective

pressure of the steam in pounds, and, further, that product by the co-efficient in the table (belonging to its driving wheels and stroke of the pistons), and this new product will be the traction of the engine in pounds.

EXAMPLE.—A locomotive engine having 5 feet 6 inches driving wheels, cylinders of 13 inches diameter by 18 inches stroke, and the effective pressure of the steam being 40 lbs. on the square inch, what is the traction?

$$(2 \times 132{\cdot}66)40 \times 0{\cdot}1736 = 1842{\cdot}39 \text{ lbs.}$$

If it be required to know the number of tons the engine is able to draw on a level, divide its traction by the friction in pounds.

If the engine is to go up inclines, then add to that friction the *gravity* in pounds due to a ton on that incline, and use this sum as a divisor for the traction; the quotient will be the number of tons the engine is capable of drawing up the incline.

In both cases the weight of the engine and its tender is included in the quotient.

Explanations.—By *effective pressure* is understood the pressure of the steam above the pressure of the atmosphere, less the number of pounds necessary to keep the engine by itself just in motion.

Friction.—The power necessary to move a mass along, which is generally taken to be on railroads equal to 10 lbs. for every ton.

Gravity.—The power to overcome the tendency of a mass or load to descend an incline, being always equal to the quotient of the product of the load and height of the incline, divided by the length of the incline.

Therefore the above engine would draw $\frac{1842{\cdot}39}{10} = 184$ tons on a level; and on an inclined plane, say as 1 in 300.

Friction = 10 lbs.

Gravity $= \frac{1 \times 2240}{300} = 7{\cdot}466$ lbs.; consequently, $\frac{1842{\cdot}39}{17{\cdot}466} = 105{\cdot}5$ tons.

If the weight of the engine with its tender be taken at 18 tons, it will draw a net gross load of 166 tons on a level, and 87·5 tons up an incline of 1 in 300.

TABLE SHEWING THE CIRCUMFERENCE OF DIFFERENT DRIVING WHEELS.

Diameter of Wheel.	Length of Circumference.	Diameter of Wheel.	Length of Circumference.
ft. in.	feet.	ft. in.	feet.
4 0	12·566	6 6	20·419
4 6	13·927	7 0	21·990
5 0	15·707	7 6	23·561
5 6	17·278	8 0	25·132
6 0	18·849		

To ascertain the amount of weight or pressure on the safety-valve of a locomotive engine.

Divide the length of the lever by the distance from its centre of motion to centre of valve, and multiply the indicated pressure on the spring-balance by the quotient, to which add the action or pressure of the lever or spring-balance; divide the sum by the area of the valve, and the quotient equals the pressure on each side of the boiler.

EXAMPLE.—Suppose an engine with valve levers of 22½ inches in length, and the distance from the pin to centre of valve 2½ inches; the action of the lever and spring balance, 5 lbs.; the indicated pressure, 50 lbs.; and the area of the valve, 7 inches: required the pressure on each square inch.

$$22{\cdot}5 \div 25 = 9, \text{ and } \frac{9 \times 50 + 5}{7} = 65 \text{ lbs. per square inch.}$$

TABLE SHEWING THE APPROXIMATE USEFUL EFFECT OF AN ORDINARY LOCOMOTIVE ENGINE AT DIFFERENT VELOCITIES.

Load in tons.	Miles per hour.	Miles per hour.	Load in tons.
25	30·90	10	250
50	25·15	12½	184
75	22·54	15	138
100	18·18	17½	106
125	15·98	20	83
150	14·29	22½	65
175	13·28	25	50
200	11·20	27½	38
225	10·77	30	28

TABLE SHEWING THE NUMBER OF REVOLUTIONS OF THE DRIVING WHEELS, OR STROKE OF THE PISTON PER MINUTE, WHILE THE ENGINE IS PERFORMING A KNOWN NUMBER OF MILES PER HOUR.

Diam. of wheel, 4 ft.	Diam. of wheel, 4 ft. 6 in.	Diam. of wheel, 5 ft.	Diam. of wheel, 5 ft. 6 in.	Diam. of wheel, 6 ft.	Diam. of wheel, 6 ft. 6 in.	Diam. of wheel, 7 ft.	Diam. of wheel, 7 ft. 6 in.	Diam. of wheel, 8 ft.	Number of miles performed per hour.
No. of revol. or strokes per min.	No. of revol. or strokes per min.	No, of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	
35·03	31·59	28·03	25·47	23·34	21·55	20	18·67	17·5	5
70·06	63·17	56·06	50·95	46·68	43·10	40	37·34	35·0	10
105·09	94·76	84·09	76·42	70·02	64·65	60	56·01	52·5	15
140·12	126·36	112·12	101·70	93·36	86·20	80	74·68	75·0	20
175·15	157·59	140·20	127·37	116·70	107·75	100	93·35	87·5	25
210·18	189·54	168·18	152·85	140·04	129·30	120	112·02	105·0	30
245·21	221·13	196·21	177·29	163·38	150·85	140	130·69	122·5	35
280·24	252·72	224·24	203·76	186·72	172·40	160	149·36	140·0	40
315·27	284·30	252·27	229·23	210·06	193·95	180	168·03	157·5	45
350·30	315·90	280·30	254·75	233·40	215·50	200	186·70	175·0	50
385·33	347·49	308·33	280·17	256·74	237·05	220	205·37	192·5	55
420·36	379·08	336·36	305·64	280·08	258·60	240	224·04	210·0	60
455·39	416·67	364·39	331·11	303·42	280·15	260	242·71	227·5	65
490·42	442·19	392·42	356·58	326·76	301·70	280	361·38	245·0	70
525·45	473·85	420·50	372·05	350·10	323·25	300	280·05	262·5	75
560·48	505·50	448·48	407·60	373·44	344·80	320	298·72	280·0	80
595·51	536·89	477·01	432·99	396·78	366·35	340	317·39	297·5	85
630·24	568·60	504·51	458·55	420·12	387·90	360	336·06	315·0	90
655·57	600·19	532·57	483·93	443·46	409·45	380	354·73	332·5	95
700·60	631·78	560·63	509 50	466·80	431·00	400	373·40	350·0	100

Note.—To find the velocity the piston is travelling at in feet per minute, multiply the number of revolutions of its driving wheels, in the table, by *twice* the length of its stroke in feet.

EXAMPLE.—What is the speed of the piston of an engine with 6-feet driving wheels and 15-inch stroke, when going at the rate of 50 miles an hour?

By means of the table—

$$233{\cdot}40 \text{ revolutions} \times \frac{2 \times 5}{4} = 583{\cdot}5 \text{ feet per minute.}$$

The number of revolutions of driving wheels are inversely as their diameters, and in direct proportion to the number of miles performed.

EXAMPLE.—*How many* revolutions have the driving wheels of

an engine to make when it is going at 95 miles an hour, their diameter being 9 feet 9 inches? According to the table, a 4-feet wheel would have to make 665·57 revolutions; therefore,

$$9{\cdot}5 : 4 :: 665{\cdot}57 : \frac{4 \times 665{\cdot}57}{9{\cdot}5} = 280 \text{ revolutions.}$$

The driving wheels of an engine make 35·03 revolutions when going at the rate of 5 miles an hour, how many will they make when going at 9 miles?

$$5 : 9 :: 35{\cdot}03 : \frac{9 \times 35{\cdot}03}{5} = 63{\cdot}05 \text{ revolutions.}$$

APPENDIX.

Table of Specific Gravities.

Substances.	Specific gravity.	Weight of a cubic inch.	Substances.	Specific gravity.	Weight of a cubic inch.
Metals.			**Metals.**		
Aluminum,	2560	·0926	Mercury, 212 deg.,	13370	·4836
Antimony,	6712	·2428	Molybdenum,	8600	·3111
Arsenic,	5763	·2084	Nickel,	8800	·3183
Barium,	470	·017	Nickel, cast,	8279	·2994
Bismuth,	9823	·3553	Osmium,	10000	·3613
Brass, copper, 84, Brass, tin, 16,	8832	·3194	Palladium,	11350	·4105
			Platinum, hammered,	20337	·7356
Brass, copper, 67, Brass, zinc, 33,	7820	·2828	Platinum, native,	16000	·5787
			Platinum, rolled,	22069	·7982
Brass, plate,	8380	·3031	Potassium, 59 deg.,	865	·0313
Brass, wire,	8214	·2972	Red lead,	8940	·324
Bronze, gun-metal,	8700	·3147	Rhodium,	10650	·3852
Boron,	2000	·0723	Ruthenium,	8600	·3111
Bromine,	3000	·1085	Selenium,	4500	·1627
Cadmium,	8650	·3129	Silicium,	...	...
Calcium,	1580	·057	Silver, pure, cast,	10474	·3788
Chromium,	5900	·2134	Silver, pure, hammered,	10511	·3802
Cinnabar,	8098	·2929	Sodium,	970	·0351
Cobalt,	8600	·3111	Steel, plates,	7806	·2823
Columbium,	6000	·217	Steel, soft,	7833	·2833
Gold, pure, cast,	19258	·6965	Steel, tempered and hardened,	7818	·2828
Gold, hammered,	19361	·7003			
Gold, 22 carats fine,	17486	·6325			
Gold, 20 carats fine,	15709	·5682	Steel, wire,	7847	·2838
Copper, cast,	8788	·3179	Strontium,	2540	·0918
Copper, plates,	8698	·3146	Tin, Cornish, hammered,	7390	·2673
Copper, wire,	8880	·3212			
Iridium,	18680	·6756	Tin, Cornish, pure,	7291	·2637
Iridium, hammered,	23000	·8319	Tellurium,	6110	·221
Iron, cast,	7207	·2607	Thalium,	11850	·4286
Iron, cast, gun-metal,	7308	·264	Titanium,	5300	·1917
Iron, hot blast,	7065	·2555	Tungsten,	17000	·6149
Iron, cold,	7218	·2611	Uranium,	10150	·3671
Iron, wrought bars,	7788	·2817	Wolfram,	7119	·2575
Iron, wrought wire,	7774	·2811	Zinc, cast,	6861	·2482
Iron, rolled plates,	7704	·2787	Zinc, rolled,	7119	·26
Lead, cast,	11352	·4106			
Lead, rolled,	11388	·4119			Cubic foot.
Lithium,	590	·0213	**Woods (Dry).**		
Manganese,	8000	·2894	Alder,	800	·50
Magnesium,	1750	·0633	Apple,	793	49·562
Mercury – 40 deg.,	15632	·5661	Ash,	845	52·812
Mercury + 32 deg.,	13598	·4918		600	43·125
Mercury, 60 deg.,	13580	·4912	Bamboo,	400	·25
			Bay,	822	51·375

Substances.	Specific gravity.	Weight of a cubic foot.
Woods (Dry).		
Beech,	852	53·25
Beech,	690	43·125
Birch,	567	35·437
Box, Brazilian,	1031	64·437
Box, Dutch,	912	57·
Box, French,	1328	83·
Bullet-wood,	928	58·
Butter-nut,	376	23·5
Campeachy,	913	57·062
Cedar,	561	35·062
Cedar, Indian,	1315	82·157
Charcoal, pine,	441	27·562
Charcoal, fresh burned	380	23·75
Charcoal, oak,	1573	98·312
Charcoal, soft-wood,	280	17·5
Charcoal, triturated,	1380	86·25
Cherry,	715	44·687
Chesnut, sweet,	610	38·125
Citron,	726	45·375
Cocoa,	1040	65·
Cork,	240	15·
Cypress,	644	40·25
Dog-wood,	756	47·25
Ebony, American,	1331	83·187
Ebony, Indian,	1209	75·562
Elder,	695	43·437
Elm,	750	35·625
	671	41·937
Filbert,	600	37·5
Fir, Norway spruce,	512	32·
Gum, blue,	843	52·687
Gum, water,	1000	62·5
Hackmatack,	592	37·
Hazel,	860	53·75
Hawthorn,	910	56·875
Hemlock,	368	23·
Hickory, pig-nut,	792	49·5
Hickory, shell-bark,	690	43·125
Holly,	760	47·5
Jasmine,	770	48·125
Juniper,	566	35·375
Lance-wood,	720	45·
Larch,	544	34·
	560	35·
Lemon,	703	43·937
Lignum-vitæ,	1333	83·312
Lime,	804	50·25
Linden,	604	37·75

Substances.	Specific gravity.	Weight of a cubic foot.
Woods (Dry).		
Locust,	728	45·5
Logwood,	913	57·062
Mahogany,	720	45·
	1063	66·437
Mahogany, Honduras,	560	35·
Mahogany, Spanish,	852	53·25
Maple,	750	46·875
Maple, bird's-eye,	576	36·
Mastic,	849	53·062
Mulberry,	561	35·062
	897	56·062
Oak, African	872	51·437
Oak, Canadian	759	54·5
Oak, Dantzic	932	47·437
Oak, English	1146	58·25
Oak, green	1170	71·625
Oak, heart, sixty years old	1260	73·125
Oak, live green,	1068	78·75
Oak, live seasoned,	860	66·75
Oak, white,	823	53·75
Orange,	705	44·062
Pear,	661	41·312
Persimmon,	710	44·375
Plum,	785	49·062
Pine, pitch,	660	41·25
Pine, red,	590	36·875
Pine, white,	554	34·625
Pine, yellow,	461	28·812
Pomegranate,	1354	84·625
Poon,	580	36·25
Poplar,	383	23·937
Poplar, white,	529	33·062
Quince,	705	44·062
Rosewood,	728	45·5
Sassafras,	482	30·125
Satinwood,	885	55·312
Spruce,	500	31·25
Sycamore,	623	38·937
Tamarack,	383	23·937
Teak, African oak,	657	41·062
	745	46·562
Walnut,	671	41·062
Walnut, black,	500	31·25
Willow,	486	30·375
	585	36·562
Yew, Dutch,	788	49·25
Yew, Spanish,	807	50·437

Substances.	Specific gravity.	Weight of a cubic foot.
Woods (well seasoned).		
Ash,	722	45·125
Beech,	624	39·
Cherry,	606	37·875
Cypress,	441	27·562
Hickory, red,	838	52·375
Mahogany, St. Domingo,	720	45·
Pine, white,	473	29·562
Pine, yellow,	541	33·812
Poplar,	587	36·687
White Oak (upland),	687	42·937
White Oak, James's River,	759	42·437
Stones, Earths, &c.		
Agate,	2590	...
Alabaster, white,	2730	170·625
Alabaster, yellow,	2699	168·687
Alum,	1714	107·125
Amber,	1078	67·375
Ambergris,	866	...
Asbestos, starry,	3073	192·062
Asphaltum,	905	56·562
	1650	103·125
Barytes, sulphate,	4000	250·
	4865	304·062
Basalts,	2740	171·25
	2864	179·
Borax,	1714	107·125
Brick,	1900	118·75
	1367	85·437
Brick, fire,	2201	137·562
Brickwork, in cement,	1800	112·50
Brick, in mortar,	1600	100·
	2000	125·
Carbon,	3500	218·75
Cement, Portland,	1300	81·25
Cement, Roman,	1560	97·25
Chalk,	1520	174·
	2784	...
Clay,	1930	120·625
Clay, with gravel,	2480	155·
Coal, Anthracite,	1436	89·75
	1640	102·5
Coal, Borneo,	1200	80·625
Coal, Cannel,	1238	77·375
	1318	82·375
Stones, Earths, &c.		
Coal, caking,	1277	79·812
Coal, cherry,	1276	79·75
Coal, Derbyshire,	1292	80·75
Coal, Lancaster,	1273	79·562
Coal, Maryland,	1355	84·687
Coal, Newcastle,	1270	79·375
Coal, Rive de Gier,	1300	81·25
Coal, Scotch,	1259	78·687
	1300	81·25
Coal, splint,	1302	81·375
Coal, Wales, mean,	1315	82·187
Coke,	1000	62·5
Coke, Nat'l Va,	746	46·64
Concrete, mean,	2000	125·
Copal,	1045	65·312
Coral, red,	2700	...
Coral, white,	2550	...
Cornelian,	2613	...
Diamond, Oriental,	3521	...
Diamond, Brazilian	3444	...
Earth, common soil,	2194	137·125
Earth, loose,	1500	93·75
Earth, moist sand,	2050	128·125
Earth, mould, fresh,	2050	128·125
Earth, rammed,	1600	100·
Earth, rough sand,	1020	120·
Earth, with gravel,	2020	126·25
Emery,	4000	250·
Flint, black,	2582	161·375
Flint, white,	2594	162·125
Fluorine,	1320	82·5
Glass, bottle,	2732	170·75
Glass, crown,	2487	155·437
Glass, flint,	2933	183·312
	3200	196·
Glass, green,	2642	165·125
Glass, optical,	3450	215·625
Glass, white,	2892	180·75
Glass, window,	2642	165·125
Garnet,	4180	...
Garnet, black,	3750	...
Granite, Egyptian red,	2654	165·875
Granite, Patapsco,	2640	165·
Granite, quincy,	2652	165·75
Granite, Scotch,	2625	164·062
Granite, Susquehanna,	2704	169·
Gravel, common,	1749	109·312
Grindstone,	2143	133·937

Substances.	Specific gravity.	Weight of a cubic foot.
STONES, EARTHS, &c.		
Gypsum, opaque,	2168	135·5
Hone, white, razor,	2876	179·75
Hornblende,	3540	221·25
Iodine,	4940	...
Jet,	1300	...
Lime, hydraulic,	2745	171·562
Lime, quick,	804	50·25
Limestone, green,	3180	198·75
Limestone, white,	3156	197·25
Magnesia, carbonate,	2400	150·
Marble, Adelaide,	2715	169·687
Marble, African,	2708	169·25
Marble, Biscayan, black,	2695	168·437
Marble, Carara,	2716	169·75
Marble, common,	2686	167·875
Marble, Egyptian,	2668	166·75
Marble, French,	2649	165·562
Marble, Italian, white,	2708	169·25
Marble, Parian,	2838	177·375
Marble, Vermont, white,	2650	165·75
Marl, mean,	1750	109·375
Mica,	2800	175·
Mortar,	1384	86·5
Mortar,	1750	109·375
Millstone,	2484	155·25
Mud,	1630	101·875
Nitre,	1900	118·75
Opal,	2114	...
Oyster-shell,	2092	130·75
Paving stone,	2416	151·
Pearl, Oriental,	2650	...
Peat,	600	37·5
	1329	83·062
Phosphorus,	1770	110·625
Plaster of Paris,	1176	73·5
Plumbago,	2100	131·25
Porphyry, red,	2765	172·812
Porcelain, China,	2300	143·75
Pumice stone,	915	57·187
Quartz,	2660	166·25
Rotten-stone,	1981	123·812
Red-lead,	8940	558·75
Resin,	1089	68·062
Rock-crystal,	2735	170·937

Substances.	Specific gravity.	Weight of a cubic foot.
STONES, EARTHS, &c.		
Ruby,	4283	...
Salt, common,	2130	133·125
Saltpetre,	2090	130·625
Sand, coarse,	1800	112·5
Sand, common,	1670	104·375
Sand, damp and loose,	1392	87·
Sand, dried and loose,	1560	97·5
Sand, dry,	1420	88·75
Sand, mortar, Fort Richard,	1659	103·66
Sand, mortar, Brooklyn,	1716	107·25
Sand, silicious,	1701	106·33
Sapphire,	3994	...
Shale,	2600	162·5
Slate,	2900	181·25
	2672	167·
Slate, purple,	2784	174·
Smalt,	2440	152·5
Stone, Bath, England,	1961	122·562
Stone, Blue Hill,	2640	165·
Stone, Bluestone, (basalt),	2625	164·062
Stone, Breakneck, N.Y.,	2704	169·
Stone, Bristol, England,	2510	156·875
Stone, Caen, Normandy,	2076	129·75
Stone, common,	2520	157·5
Stone, Craigleth, England,	2316	144·75
Stone, Kentish rag, England,	2651	165·687
Stone, Kip's Bay, N.Y.,	2759	172·
Stone, Norfolk, Parlt. House,	2304	144·
Stone, Portland, England,	2368	148·
Stone, sandstone, mean,	2200	137·5
Stone, Sydney,	2237	139·812
Stone, Staten Island, N.Y.,	2976	186·

Substances.	Specific gravity.	Weight of a cubic foot.
STONES, EARTHS, &c.		
Stone, Sullivan Co., N.Y.,	2688	168·
Schorl,	3170	198·125
Spar, calcareous,	2735	170·937
Spar, Feld, blue,	2693	168·312
Spar, Feld, green,	2704	169·
Spar, Fluor,	3400	215·5
Stalactite,	2415	150·937
Sulphur, native,	2033	127·062
Talc, mean,	2500	156·25
Tale, black,	2900	181·25
Tile,	1815	113·437
Topaz, Oriental,	4011	...
Trap,	2720	170·
Turquoise,	2750	...
MISCELLANEOUS.		
Asphaltum,	905	56·562
	1650	103·125
Atmospheric air,	001205	·07529
Beeswax,	965	60·312
Butter,	942	58·875
Camphor,	988	61·75
Caoutchouc,	903	56·437
Egg,	1090	...
Fat of beef,	923	57·687
Fat of hogs,	936	58·5
Fat of mutton,	923	57·687
Gamboge,	1222	...
Gum Arabic,	1452	90·75
Gunpowder, loose,	900	56·25
Gunpowder, shaken,	1000	62·5
Gunpowder, solid,	1550	96·875
	1800	112·5
Gutta-percha,	980	61·25
Horn,	1689	105·562
Ice at 32°,	920	57·5
Indigo,	1009	63·062
Isinglass,	1111	69·437
Ivory,	1825	114·062
Lard,	947	59·187
Mastic,	1074	67·125
Myrrh,	1360	85·
Opium,	1336	83·5
Soap, Castile,	1071	56·937

Substances.	Specific gravity.	Weight of a cubic foot.
MISCELLANEOUS.		
Spermaceti,	943	58·937
Starch,	950	59·375
Sugar,	1606	100·375
Sugar 66,	1326	82·875
Tallow,	941	58·812
Wax,	964	60·25
	970	60·625
LIQUIDS.		
Acid, acetic,	1062	66·375
Acid, benzoic,	667	41·687
Acid, citric,	1034	64·625
Acid, concentrated,	1521	95·062
Acid, fluoric,	1500	93·75
Acid, muriatic,	1200	75·
Acid, nitric,	1217	76·062
Acid, phosphoric,	1558	97·375
Acid, phosphoric, solid,	2800	175·
Acid, sulphuric,	1849	115·562
Alcohol, pure, 60°,	794	49·622
Alcohol 95 per cent.,	816	51·
Alcohol 80 per cent.,	863	53·937
Alcohol 50 per cent.,	934	58·375
Alcohol 40 per cent.,	951	59·437
Alcohol 25 per cent.,	970	60·625
Alcohol 10 per cent.,	986	61·625
Alcohol 5 per cent.,	992	62·
Alcohol proof spirit 50 per cent., Alcohol 60°,	934	58·375
Alcohol proof spirit 50 per cent., Alcohol 80°,	875	54·687
Ammonia 27·9 per cent.,	891	55.687
Aquafortis, double,	1300	81·25
Aquafortis, single,	1200	75·
Beer,	1034	64·625
Bitumen, liquid,	848	53·
Blood (human),	1054	65·875
Brandy $\frac{4}{5}$ or 5 of spirit,	924	57·75
Cider,	1018	63·625
Ether, acetic,	866	54·125
Ether, muriatic,	845	52·820

Substances.	Specific gravity.	Weight of a cubic foot.	Substances.	Specific gravity.	Weight of a cubic foot.
LIQUIDS.			LIQUIDS.		
Ether, sulphuric,	715	44·687	Spirit, rectified,	824	51·5
Honey,	1450	90·625	Tar,	1015	63·437
Milk,	1032	64·5	Vinegar,	1080	67·5
Oil, anise-seed,	986	61·625	Water, Dead Sea,	1240	77·5
Oil, codfish,	923	57·687	Water, 60°	999	62·449
Oil, cotton-seed,	...	...	Water, 212°,	957	59·812
Linseed,	940	58·75	Water, distilled, 30°,	998	62·379
Oil, naphtha,	848	53·	Water, Mediterranean	1029	64·312
Oil, olive,	915	57·187	Water, rain,	1000	62·5
Oil, palm,	960	60·562	Water, sea,	1026	64·125
Oil, petroleum,	878	54·875	Wine, Burgundy,	992	62·
Oil, rape,	914	57·125	Wine, Champagne,	997	64·375
Oil, sunflower,	926	57·875	Wine, Maderia,	1038	62·312
Oil, turpentine,	870	54·375	Wine, Port,	997	62·312
Oil, whale,	923	57·687			

Compression of the following fluids under a pressure of 15 lbs. per square inch:—

Alcohol,	·0000216	Mercury,	·00000265
Ether,	·00006158	Water,	·00004663

WEIGHT OF FLAT ROLLED IRON, FROM $\frac{1}{2} \times \frac{1}{8}$ INCH TO $5\frac{3}{4} \times 6$ INCHES, 1 FOOT IN LENGTH.

Thickness.	Weight.	Thickness.	Weight.	Thickness.	Weight.	Thickness.	Weight.
Inches.	Lbs.	Inches.	Lbs.	Inches.	Lbs.	Inches.	Lbs.
$\frac{1}{8}$	...	$\frac{7}{8}$	3·326	$1\frac{3}{8}$	6·97	$1\frac{1}{8}$	7·129
$\frac{1}{8}$	·211	1·	3·802	$1\frac{3}{8}$	...	$1\frac{1}{8}$	7·921
$\frac{1}{8}$	·422	$1\frac{1}{4}$	...	$\frac{1}{8}$	·686	$1\frac{3}{8}$	8·713
$\frac{1}{8}$	·634	$\frac{1}{8}$	·528	$\frac{1}{8}$	1·372	$1\frac{1}{2}$	9·505
$\frac{1}{8}$	...	$\frac{1}{8}$	1·056	$\frac{1}{8}$	2·059	$1\frac{5}{8}$	10·297
$\frac{1}{8}$	·264	$\frac{3}{8}$	1·584	$\frac{1}{2}$	2·746	$1\frac{3}{4}$	11·089
$\frac{1}{8}$	·528	$\frac{1}{2}$	2·112	$\frac{5}{8}$	3·432	2·	...
$\frac{1}{8}$	·792	$1\frac{1}{4}$	...	$\frac{3}{4}$	4·119	$\frac{1}{8}$	·845
$\frac{1}{8}$	1·056	$\frac{5}{8}$	2·64	$\frac{7}{8}$	4·805	$\frac{1}{8}$	1·689
$\frac{3}{4}$	...	$\frac{3}{4}$	3·168	1·	5·492	$\frac{3}{8}$	2·534
$\frac{1}{8}$	·316	$\frac{7}{8}$	3·696	$1\frac{1}{8}$	6·178	$\frac{1}{2}$	3·379
$\frac{1}{8}$	·633	1·	4·224	$1\frac{1}{4}$	6·864	$\frac{5}{8}$	4·224
$\frac{1}{8}$	·95	$1\frac{1}{8}$	4·752	$1\frac{3}{8}$	7·551	$\frac{3}{4}$	5·069
$\frac{1}{8}$	1·265	$1\frac{3}{8}$	...	$1\frac{1}{2}$	8·237	$\frac{7}{8}$	5·914
$\frac{1}{8}$	1·584	$\frac{1}{8}$	·58	$1\frac{3}{4}$	...	1·	6·758
$\frac{1}{8}$	...	$\frac{1}{8}$	1·161	$\frac{1}{8}$	·739	$1\frac{1}{8}$	7·604
$\frac{1}{8}$	·369	$\frac{3}{8}$	1·742	$\frac{1}{4}$	1·479	$1\frac{1}{4}$	8·448
$\frac{1}{4}$	·738	$\frac{1}{2}$	2·325	$\frac{3}{8}$	2·218	$1\frac{3}{8}$	9·294
$\frac{3}{8}$	1·108	$\frac{5}{8}$	2·904	$\frac{1}{2}$	2·957	$1\frac{1}{2}$	10·138
$\frac{1}{2}$	1·477	$\frac{3}{4}$	3·484	$\frac{5}{8}$	3·696	$1\frac{5}{8}$	10·983
$\frac{5}{8}$	1·846	$\frac{7}{8}$	4·065	$\frac{3}{4}$	4·435	$1\frac{3}{4}$	11·828
$\frac{3}{4}$	2·217	1·	4·646	$\frac{7}{8}$	5·178	$1\frac{7}{8}$	12·673
1·	...	$1\frac{1}{8}$	5·227	1·	5·914	$2\frac{1}{8}$	...
$\frac{1}{8}$	·422	$1\frac{1}{4}$	5·808	$1\frac{1}{8}$	6·653	$\frac{1}{8}$	·898
$\frac{1}{4}$	·845	$1\frac{3}{8}$	6·389	$1\frac{1}{4}$	7·393	$\frac{1}{4}$	1·795
$\frac{3}{8}$	1·267	$1\frac{1}{2}$	...	$1\frac{3}{8}$	8·132	$\frac{3}{8}$	2·693
$\frac{1}{2}$	1·69	$\frac{1}{8}$	·633	$1\frac{1}{2}$	8·871	$\frac{1}{2}$	3·591
$\frac{5}{8}$	2·112	$\frac{1}{4}$	1·266	$1\frac{5}{8}$	9·61	$\frac{5}{8}$	4·488
$\frac{3}{4}$	2·534	$\frac{3}{8}$	1·9	$1\frac{3}{4}$	...	$\frac{3}{4}$	5·386
$\frac{7}{8}$	2·956	$1\frac{1}{2}$	...	$\frac{1}{8}$	·792	$\frac{7}{8}$	6·283
$1\frac{1}{8}$	...	$\frac{1}{2}$	2·535	$\frac{1}{4}$	1·584	$2\frac{1}{8}$	...
$\frac{1}{8}$	·475	$\frac{5}{8}$	3·168	$\frac{3}{8}$	2·376	1·	7·181
$\frac{1}{4}$	·95	$\frac{3}{4}$	3·802	$\frac{1}{2}$	3·168	$1\frac{1}{8}$	8·079
$\frac{3}{8}$	1·425	$\frac{7}{8}$	4·435	$\frac{5}{8}$	3·96	$1\frac{1}{4}$	8·977
$\frac{1}{2}$	1·901	1·	5·069	$\frac{3}{4}$	4·752	$1\frac{3}{8}$	9·874
$\frac{5}{8}$	2·375	$1\frac{1}{8}$	5·703	$\frac{7}{8}$	5·544	$1\frac{1}{2}$	10·772
$\frac{3}{4}$	2·85	$1\frac{1}{4}$	6·337	1·	6·336	$1\frac{5}{8}$	11·67

Thickness.	Weight.	Thickness.	Weight.	Thickness.	Weight.	Thickness.	Weight.
Inches.	Lbs.	Inches.	Lbs.	Inches.	Lbs.	Inches.	Lbs.
1·3/4	12·567	·1/8	1·056	2·3/4	...	2·1/4	21·861
1·7/8	13·465	·1/4	2·112	·1/8	1·162	2·3/8	23·076
2·	14·362	·3/8	3·168	·1/4	2·323	2·1/2	24·29
2·1/4	...	·1/2	4·224	·3/8	3·485	2·5/8	25·505
·1/8	95	·5/8	5·28	·1/2	4·647	2·3/4	26·719
·1/4	1·9	2·1/2	...	·5/8	5·808	3·	...
·3/8	2·851	·3/4	6·336	·3/4	6·97	·1/8	1·267
·1/2	3·802	·7/8	7·392	·7/8	8·132	·1/4	2·535
·5/8	4·752	1·	8·448	1·	9·294	·3/8	3·802
·3/4	5·703	1·1/8	9·504	1·1/8	10·455	·1/2	5·069
·7/8	6·653	1·1/4	10·56	1·1/4	11·617	·5/8	6·337
1·	7·604	1·3/8	11·616	1·3/8	12·779	·3/4	7·604
1·1/8	8·554	1·1/2	12·672	1·1/2	13·94	·7/8	8·871
1·1/4	9·505	1·5/8	13·728	1·5/8	15·102	1·	10·138
1·3/8	10·455	1·3/4	14·784	1·3/4	16·264	1·1/8	11·406
1·1/2	11·406	1·7/8	15·84	1·7/8	17·425	1·1/4	12·673
1·5/8	12·356	2·	16·896	2·	18·587	1·3/8	13·94
1·3/4	13·307	2·1/8	17·952	2·1/8	19·749	1·1/2	15·208
1·7/8	14·259	2·1/4	19·008	2·1/4	...	1·5/8	16·475
2·	15·208	2·3/8	20·064	2·1/4	20·91	1·3/4	17·742
2·1/8	16·158	2·1/2	...	2·3/8	22·072	1·7/8	19·01
2·3/8	...	·1/8	1·109	2·1/2	23·234	2·	20·277
·1/8	1·003	·1/4	2·218	2·5/8	24·395	2·1/4	22·811
·1/4	2·006	·3/8	3·327	2·7/8	...	2·1/2	25·346
·3/8	3·009	·1/2	4·436	·1/8	1·215	2·3/4	27·881
·1/2	4·013	·5/8	5·545	·1/4	2·429	3·1/4	...
·5/8	5·016	·3/4	6·654	·3/8	3·644	·1/8	1·373
·3/4	6·019	·7/8	7·763	·1/2	4·858	·1/4	2·746
·7/8	7·022	1·	8·872	·5/8	6·072	·3/8	4·119
1·	8·025	1·1/8	9·981	·3/4	7·287	·1/2	5·492
1·1/8	9·028	1·1/4	11·09	·7/8	8·502	3·1/4	...
1·1/4	10·032	1·3/8	12·199	1·	9·716	·5/8	6·865
1·3/8	11·035	1·1/2	13·308	1·1/8	10·931	·3/4	8·237
1·1/2	12·038	1·5/8	14·417	1·1/4	12·145	·7/8	9·61
1·5/8	13·042	1·3/4	15·526	1·3/8	13·36	1·	10·983
1·3/4	14·045	1·7/8	16·635	1·1/2	14·574	1·1/8	12·356
1·7/8	15·048	2·	17·744	1·5/8	15·789	1·1/4	13·73
2·	16·051	2·1/8	18·853	1·3/4	17·003	1·3/8	15·102
2·1/8	17·054	2·1/4	19·962	1·7/8	18·218	1·1/2	16·475
2·1/4	18·057	2·3/8	21·071	2·	19·432	1·5/8	17·848
2·1/2	...	2·1/2	22·18	2·1/8	20·647	1·3/4	19·221

Thickness.	Weight.	Thickness.	Weight.	Thickness.	Weight.	Thickness.	Weight.
Inches.	Lbs.	Inches.	Lbs.	Inches.	Lbs.	Inches.	Lbs.
1·7/8	20·594	1·5/8	20·594	3·	43·088	4·1/4	68·223
2·	21·967	3·3/4	...	3·1/4	46·679	4·1/2	72·235
2·1/4	24·712	1·3/4	22·178	3·1/2	50·269	5·	...
2·1/2	27·458	1·7/8	23·762	3·3/4	53·86	·1/4	4·224
2·3/4	30·204	2·	25·346	4·	57·45	·1/2	8·449
3·	32·95	2·1/4	28·514	4·1/8	...	·3/4	12·673
3·1/8	...	2·1/2	31·682	·1/4	3·802	1·	16·897
·1/8	1·479	2·3/4	34·851	·1/2	7·604	1·1/4	21·122
·1/4	2·957	3·	38·019	·3/4	11·406	1·1/2	25·346
·3/8	4·436	3·1/4	41·187	1·	15·208	1·3/4	29·57
·1/2	5·914	3·1/2	44·355	1·1/4	19·01	2·	33·795
·5/8	7·393	4·	...	1·1/2	22·812	2·1/4	38·019
·3/4	8·871	·1/8	1·69	1·3/4	26·614	2·1/2	42·243
·7/8	10·35	·1/4	3·38	4·1/2	...	2·3/4	46·468
1·	11·828	·1/2	6·759	2·	30·415	3·	50·692
1·1/8	13·307	·3/4	10·138	2·1/4	34·217	3·1/4	54·916
1·1/4	14·785	1·	13·518	2·1/2	38·019	3·1/2	59·14
1·3/8	16·264	1·1/4	16·897	2·3/4	41·82	3·3/4	63·365
1·1/2	17·742	1·1/2	20·277	3·	45·623	4·	67·589
1·5/8	19·221	1·3/4	23·656	3·1/4	49·425	4·1/4	71·813
1·3/4	20·699	2·	27·036	3.1/2	53·226	4·1/2	76·038
1·7/8	22·178	2·1/4	30·415	3·3/4	57·028	4·3/4	80·262
2·	23·656	2·1/2	33·795	4·	60·83	5·1/4	...
2·1/4	26·613	2·3/4	37·174	4·1/4	64·632	·1/4	4·436
2·1/2	29·57	3·	40·554	4·1/8	...	5·1/4	...
2·3/4	32·527	3·1/4	43·933	·1/4	4·013	·3/4	8·871
3·	35·485	3·1/2	47·313	·1/8	8·026	·3/4	13·307
3·1/4	38·441	3·3/4	50·692	·3/4	12·036	1·	17·742
3·3/8	...	4·1/4	...	1·	16·052	1·1/4	22·178
·1/8	1·584	·1/8	1·795	1·1/4	20·066	1·1/2	26·613
·1/4	3·168	·1/4	3·591	1·1/2	24·079	1·3/4	31·049
·3/8	4·752	·1/2	7·181	1·3/4	28·092	2·	35·484
·1/2	6·336	·3/4	10·772	2·	32·105	2·1/4	39·92
·5/8	7·921	1·	14·364	2·1/2	36·118	2·1/2	44·355
·3/4	9·505	1·1/4	17·953	2·1/2	40·131	2·3/4	48·791
·7/8	11·089	1·1/2	21·544	2·3/4	44·144	3·	53·226
1·	12·673	1·3/4	25·135	3·	48·157	3·1/4	57·662
1·1/8	14·257	2·	28·725	3·1/4	52·17	3·1/2	62·097
1·1/4	15·841	2·1/4	32·316	3·1/2	56·184	3·3/4	66·533
1·3/8	17·425	2·1/2	35·907	3·3/4	60·197	4·	70·968
1·1/2	19·009	2·3/4	39·497	4·	64·21	4·1/4	75·404

Thickness.	Weight.	Thickness.	Weight.	Thickness.	Weight.	Thickness.	Weight.
Inches.	Lb.	Inches.	Lbs.	Inches.	Lbs.	Inches.	Lbs.
4·1/16	79·839	2·1/16	46·468	·1/16	9·716	3·1/16	68·012
4·1/8	84·275	2·1/8	51·114	·1/8	14·574	5·1/16	...
5·	88·71	3·	55·761	1·	19·432	3·1/8	72·87
5·1/16	...	3·1/16	60·408	1·1/16	24·29	4·	77·728
·1/8	4·647	3·1/8	65·055	1·1/8	29·148	4·1/8	82·585
·3/16	9·294	3·3/16	69·701	1·3/16	34·006	5·1/16	...
·1/4	13·94	4·	74·348	2·	38·864	4·1/16	87·443
1·	18·587	4·1/16	78·995	2·1/16	43·722	4·1/8	92·301
1·1/16	23·234	4·1/8	83·642	2·1/8	48·58	5·	97·159
1·1/8	27·881	4·3/16	88·288	2·3/16	53·437	5·1/4	...
1·3/16	32·527	5·	92·935	5·1/4	...	5·1/16	102·017
2·	37·174	5·1/16	9·582	3·	58·296	5·1/8	106·876
2·1/16	41·821	5·1/8	97·159	3·1/16	63·154	6·	116·592

N.B.—For computation, see "Round Rolled Iron."

Weight of Cast-Iron Pipes of Different Thicknesses, from 1 Inch to 36 Inches Diameter.

One foot in length.

Diam.	Thick-ness.	Weight.	Diam.	Thick-ness.	Weight.	Diam.	Thick-ness.	Weight.
Ins.	Inches.	Lbs.	Ins.	Inches.	Lbs.	Ins.	Inches.	Lbs.
1·	·¼	3·06	4·⅛	·¾	38·58	9·	·5	58·92
	·⅝	5·05	4·¾	·1	25·7		·⅝	71·7
1·¼	·¼	3·67		·5	32·91		·⅞	84·7
	·⅝	6·		·⅝	40·43	1·		97·98
1·½	·⅜	6·89	5·	·1	26·94	9·½	·1	48·98
	·½	9·8		·5	34·34		·5	62·02
1·¾	·⅜	7·8		·⅝	42·28		·⅝	75·32
	·½	11·04	5·½	·½	29·4		·⅞	88·98
2·	·⅜	8·74		·5	37·44	1·		102·9
	·½	12·23		·⅝	45·94	10·	·1	51·46
2·¼	·⅜	9·65	6·	·½	31·82		·5	65·08
	·½	13·48		·5	40·56		·⅝	78·99
2·½	·⅜	10·57	6·	·⅝	49·6		·⅞	93·24
	·½	14·66		·⅞	58·96	1·		108·84
	·5	19·05	6·½	·½	34·32	10·½	·1	53·88
2·¾	·⅜	11·54		·5	43·68		·5	68·14
	·½	15·91		·⅝	53·3		·⅝	82·68
	·5	20·59		·⅞	63·18		·⅞	97·44
3.	·⅜	12·28	7·	·½	36·66	1·		112·68
	·½	17·15		·5	46·8	11·	·1	56·34
	·⅝	22·15		·⅝	56·96		·5	71.19
	·¾	27·56		·⅞	67·66		·⅝	86·4
3·¼	·½	18·04	1·		78·39		·⅞	101·83
	·⅝	23·72	7·½	·½	39·22	1·		117·6
	·¾	29·64		·5	49·92	11·½	·1	58·82
3·½	·½	19·66		·⅝	60·48		·5	74·28
	·⅝	25·27		·⅞	71·76		·⅝	90·06
	·¾	31·2	1·		83·28		·⅞	106·14
3·¾	·½	20·9	8·	·½	41·64	1·		122·62
	·⅝	26·83		·5	52·68	12·	·1	61·26
	·¾	33·07		·⅝	64·27		·5	77·36
4·	·½	22·05		·⅞	76·12		·⅝	93·7
	·5	20·28	1·		88·2		·⅞	110·48
	·¾	34·94	8·½	·½	44·11	1·		127·42
4·¼	·½	23·35		·5	56·16	12·½	·1	63·7
	·5	29·85		·⅝	68·		·5	80·4
	·¾	36·73		·⅞	80·5		·⅝	97·4
4·½	·½	24·49	1·		93·28		·⅞	114·72
	·⅝	31·4	9·	·½	46·5		1·	132·35

Diam.	Thickness.	Weight.	Diam.	Thickness.	Weight.	Diam.	Thickness.	Weight.
Ins.	Inches.	Lbs.	Ins.	Inches.	Lbs.	Ins.	Inches.	Lbs.
13·	·½	66·14	17·	·⅝	130·48	26·	1·	264·66
	·⅝	83·46		·⅞	153·3	27·	·¾	204·04
	·¾	101·08		1·	176·58		·⅞	239·08
	·⅞	118·97	17·½	·½	88·23		1·	274·56
	1·	137·28		·⅝	111·06	28·	·¾	211·32
13·½	·½	68·364		·¾	134·16		·⅞	247·62
	·⅝	86·55		·⅞	157·59		1·	284·28
	·¾	104·76		1·	181·33		·¾	218·7
	·⅞	123·3	18·	·⅝	114·1		·⅞	256·2
	1·	142·16		·¾	137·84		1·	294·02
14·	·½	71·07		·⅞	161·9		·¾	226·2
	·⅝	89·61		1·	186·24		·⅞	264·79
	·¾	108·46	19·	·⅝	120·24	30·	1·	303·86
	·⅞	127·6		·¾	145·2		1.⅛	343·2
	1·	147·03		·⅞	170·47	31·	·¾	233·4
14·½	·½	73·72		1·	195·92		·⅞	273·4
	·⅝	92·66	20·	·⅝	126·33		1·	313·68
	·¾	112·1		·¾	152·53		1·⅛	354·24
	·⅞	131·86		·⅞	179·02	32.	·¾	240·76
	1·	151·92		1·	205·8		·⅞	281·94
15·	·½	75·96	21·	·⅝	132·5		1.	323·49
	·⅝	95·72		·¾	159·84		1·⅛	365·29
	·¾	115·78		·⅞	187·6	33·	·¾	248·1
	·⅞	136·15		1·	215·52		·⅞	290·5
	1·	156·82	22·	·⅝	138·6		1·	333·24
15·½	·½	78·4		·¾	167·24		1·⅛	376·26
	·⅝	98·78		·⅞	196·46		1·¼	420·77
	·¾	119·48		1·	225·38	34.	·¾	255·45
	·⅞	140·4	23·	·⅝	144·77		·⅞	298·88
	1·	161·82		·¾	174·62		1·	342·88
16·	·½	80·87		·⅞	204·78		1·⅛	387·13
	·⅝	101·82		1·	235·28		1·¼	431·76
	·¾	123·14	24·	·⅝	150·85	35·	·¾	262·7
	·⅞	144·76		·¾	181·92		·⅞	307·62
	1·	166·6		·⅞	213·28		1·	352·86
16·½	·½	83·3		1·	245·08		1·⅛	398·1
	·⅝	104·82	25·	·⅝	156·97		1·¼	443·96
	·¾	126·79		·¾	189·28	36·	·¾	270·18
	·⅞	149·02		·⅞	221·94		·⅞	316·36
	1·	171·6		1·	254·86		1·	362·86
17·	·½	85·73	26·	·¾	196·92		1·⅛	409·34
	·⅝	107·96		·⅞	230·56		1·¼	456·46

Note.—These weights do not include any allowance for Spigot and Faucet ends.

TABLE OF THE SIDES OF SQUARES, EQUAL IN AREA TO A CIRCLE OF ANY DIAMETER, FROM 1 TO 100.

Diam.	Side of square.	Diam.	Side of square.	Diam.	Side of square.	Diam.	Side of square.
1·	·8862	11·	9·7485	21·	18·6108	31·	27·473
·¼	1·1078	·¼	9·97	·¼	18·8323	·¼	27·6946
·½	1·3293	·½	10·1916	·½	19·0539	·½	27·9161
·¾	1·5509	·¾	10·4132	·¾	19·2754	·¾	28·1377
2·	1·7724	12·	10·6347	22·	19·497	32·	28·3593
·¼	1·994	·¼	10·8563	·¼	19·7185	·¼	28·5808
·½	2·2156	·½	11·0778	·½	19·9401	·½	28·8024
·¾	2·4371	·¾	11·2994	·¾	20·1617	·¾	29·0239
3·	2·6587	13·	11·5209	23·	20·3832	33·	29·2455
·¼	2·8802	·¼	11·7425	·¼	20·6048	·¼	29·467
·½	3·1018	·½	11·9641	·½	20·8263	·½	29·6886
·¾	3·3233	·¾	12·1856	·¾	21·0479	·¾	29·9102
4·	3·5449	14·	12·4072	24·	21·2694	34·	30·1317
·¼	3·7665	·¼	12·6287	·¼	21·491	·¼	30·3533
·½	3·988	·½	12·8503	·½	21·7126	·½	30·5748
·¾	4·2096	·¾	13·0718	·¾	21·9341	·¾	30·7964
5·	4·4311	15·	13·2934	25·	22·1557	35·	31·0179
·¼	4·6227	·¼	13·515	·¼	22·3772	·¼	31·2395
·½	4·8742	·½	13·7365	·½	22·5988	·½	31·4611
·¾	5·0958	·¾	13·9581	·¾	22·8203	·¾	31·6826
6·	5·3174	16·	14·1796	26·	23·0419	36·	31·9042
·¼	5·5389	·¼	14·4012	·¼	23·2634	·¼	32·1257
·½	5·7605	·½	14·6227	·½	23·485	·½	32·3473
·¾	5·982	·¾	14·8443	·¾	23·7066	·¾	32·5688
7·	6·2036	17·	15·0659	27·	23·9281	37·	32·7904
·¼	6·4251	·¼	15·2874	·¼	24·1497	·¼	33·0112
·½	6·6467	·½	15·509	·½	24·3712	·½	33·2335
·¾	6·8683	·¾	15·7305	·¾	24·5928	·¾	33·4551
8·	7·0898	18.	15·9231	28·	24·8144	38·	33·6766
·¼	7·3114	·¼	16·1736	·¼	25·0359	·¼	33·8982
·½	7·5329	·½	16·3952	·½	25·2575	·½	34·1197
·¾	7·7545	·¾	16·6168	·¾	25·479	·¾	34·3413
9·	7·976	19·	16·8383	29·	25·7006	39·	34·5628
·¼	8·1976	·¼	17·0599	·¼	25·9221	·¼	34·7884
·½	8·4192	·½	17·2814	·½	26·1437	·½	35·006
·¾	8·6407	·¾	17·503	·¾	26·3653	·¾	35·2275
10·	8·8623	20·	17·7245	30·	26·5868	40·	35·4491
·¼	9·0838	·¼	17·9461	·¼	26·8084	·¼	35·6706
·½	9·3054	·½	18·1677	·½	27·0299	·½	35·8922
·¾	*9·5269*	·¾	18·3892	·¾	27·2515	·¾	36·1137

Diam.	Side of square.	Diam.	Side of square	Diam.	Side of square.	Diam.	Side of square.
41·	36·3353	·¾	45·8622	·½	55·3892	·¼	64·9161
·¼	36·5569	52·	46·0838	·¾	55·6107	·½	65·1377
·½	36·7784	·¼	46·3054	63·	55·8323	·¾	65·3592
·¾	37·	·½	46·5269	·¼	56·0538	74·	65·5808
42·	37·2215	·¾	46·7485	·½	56·2754	·¼	65·8023
·¼	37·4431	53·	46·97	·¾	56·497	·½	66·0239
·½	37·6646	·¼	47·1916	64·	56·7185	·¾	66·2455
·¾	37·8862	·½	47·4131	·¼	56·9401	75·	66·467
43·	38·1078	·¾	47·6347	·½	57·1616	·¼	66·6886
·¼	38·3293	54·	47·8562	·¾	57·3832	·½	66·9104
·½	38·5509	·¼	48·0778	65·	57·6047	·¾	67·1317
·¾	38·7724	·½	48·2994	·¼	57·8263	76·	67·3532
44·	38·994	·¾	48·5209	·½	58·0479	·¼	67·5748
·¼	39·2155	55·	48·7425	·¾	58·2694	·½	67·7964
·½	39·4371	·¼	48·964	66·	58·491	·¾	68·0179
·¾	39·6587	·½	49·1856	·¼	58·7125	77·	68·2395
45·	39·8802	·¾	49·4070	·½	58·9341	·¼	68·461
·¼	40·1018	56·	49·6287	·¾	59·1556	·½	68·6826
·½	40·3233	·¼	49·8503	67·	59·3772	·¾	68·9041
·¾	40·5449	·½	50·0718	·¼	59·5988	78·	69·1257
46·	40·7664	·¾	50·2934	·½	59·8203	·¼	69·3473
·¼	40·988	57·	50·5149	·¾	60·0419	·½	69·5688
·½	41·2096	·¼	50·7365	68·	60·2634	·¾	69·7904
·¾	41·4311	·½	50·958	·¼	60·485	79·	70·0119
47·	41·6527	·¾	51·1796	·½	60·7065	·¼	70·2335
·¼	41·8742	58·	51·4012	·¾	60·9281	·½	70·455
·½	42·0958	·¼	51·6227	69·	61·1497	·¾	70·6766
·¾	42·3173	·½	51·8443	·¼	61·3712	80·	70·8981
48·	42·5839	·¾	52·0658	·½	61·5928	·¼	71·1197
·¼	42·7604	59·	52·2874	·¾	61·8143	·½	71·3413
·½	42·982	·¼	52·5089	70·	62·0359	·¾	71·5628
·¾	43·2036	·½	52·7305	·¼	62·2574	81·	71·7844
49·	43·4251	·¾	52·9521	·½	62·479	·¼	72·0059
·¼	43·6467	60·	53·1736	·¾	62·7006	·½	72·2275
·½	43·8682	·¼	53·3952	71·	62·9221	·¾	72·4491
·¾	44·0898	·½	53·6167	·¼	63·1437	82·	72·6706
50·	44·3113	·¾	53·8383	·½	63·3652	·¼	72·8921
·¼	44·5329	61·	54·0598	·¾	63·5868	·½	73·1137
·½	44·7545	·¼	54·2814	72·	63·8083	·¾	73·3353
·¾	44·976	·½	54·503	·¼	64·0299	83·	73·5568
51·	*45·1976*	·¾	54·7245	·½	64·2514	·¼	73·7784
·¼	*45·4191*	62·	54·9461	·¾	64·4730	·½	73·9999
·½	*45·6407*	·¼	55·1676	73·	64·6946	·¾	74·2215

Diam.	Side of square.	Diam.	Side of square.	Diam.	Side of square.	Diam.	Side of square.
84·	74·4431	·¼	78·2095	·¼	81·976	·¾	85·7425
·¼	74·6647	·½	78·4316	·½	82·1975	97·	85·9646
·½	74·8862	·¾	78·6526	93·	82·4191	·¼	86·185
·¾	75·1077	89·	78·8742	·¼	82·6407	·½	86·4071
85·	75·3293	·¼	79·0957	·½	82·8622	·¾	86·6289
·¼	75·5508	·½	79·3173	·¾	83·0838	98	86·8502
·½	75·7724	·¾	79·5389	94·	83·3053	·¼	87·0718
·¾	75·9934	90·	79·7604	·¼	83·5269	·½	87·2933
86·	76·2155	·¼	79·982	·½	83·7484	·¾	87·5449
·¼	76·4371	·½	80·2035	·¾	83·970	99·	87·7364
·½	76·6586	·¾	80·4251	95·	84·1916	·¼	87·958
·¾	76·8802	91·	80·6467	·¼	84·4131	·½	88·1796
87·	77·1017	·¼	80·8682	·½	84·6347	·¾	88·4011
·¼	77·3233	·½	81·0898	·¾	84·8562	100·	88·6227
·½	77·5449	·¾	81·3113	96·	85·0778	·¼	88·8442
·¾	77·7664	92·	81·5329	·¼	85·2993	·½	89·0658
88·	77·988	·¼	81·7544	·½	85·5209	·¾	89·2874

To compute the length of the side of a square, equal in area to a circle of any diameter.

Rule.—Multiply the diameter of the circle by ·8862, and the product will equal the side of an equal square.

Table of Squares, Cubes, Reciprocals, and Common Logarithms of Numbers.

No.	Square.	Cube.	Reciprocal.	C. Log.
101	1 02 01	1 030 301	·009900990	00432
102	1 04 04	1 061 208	·009803922	00860
103	... 1 06 09	... 1 092 727	.....·009708738	01284
104	1 08 16	1 124 864	·009615385	01703
105	1 10 25	1 157 625	·009523810	02119
106	... 1 12 36	... 1 191 016	.....·009433962	02531
107	1 14 49	1 225 043	·009345794	02938
108	1 16 64	1 259 712	·009259259	03342
109	... 1 18 81	... 1 295 029	.....·009174312	03743
110	1 21 00	1 331 000	·009090909	04139
111	1 23 21	1 367 631	·009009009	04532
112	... 1 25 44	... 1 404 928	.....·008928571	04922
113	1 27 69	1 442 897	·008849558	05308
114	1 29 96	1 481 544	·008771930	05690
115	... 1 32 25	... 1 520 875	.....·008695652	06070
116	1 34 56	1 560 896	·008620690	06446
117	1 36 89	1 601 613	·008547009	06819
118	... 1 39 24	... 1 643 032	.....·008474576	07188
119	1 41 61	1 685 159	·008403361	07555
120	1 44 00	1 728 000	·008333333	07918
121	... 1 46 41	... 1 771 561	.....·008264463	08279
122	1 48 84	1 815 848	·008196721	08636
123	1 51 29	1 860 867	·008130081	08991
124	... 1 53 76	... 1 906 624	.....·008064516	09342
125	1 56 25	1 953 125	·008000000	09691
126	1 58 76	2 000 376	·007936508	10037
127	... 1 61 29	... 2 048 383	.....·007874016	10380
128	1 63 84	2 097 152	·007812500	10721
129	1 66 41	2 146 689	·007751938	11059
130	... 1 69 00	... 2 197 000	.....·007692308	11394
131	1 71 61	2 248 091	·007633588	11727
132	1 74 24	2 299 968	·007575758	12057
133	... 1 76 89	... 2 352 637	.....·007518797	12385
134	1 79 56	2 406 104	·007462687	12710
135	1 82 25	2 460 375	·007407407	13033
136	... 1 84 96	... 2 515 456	.....·007352941	13354
137	1 87 69	2 571 353	·007299270	13672
138	1 90 44	2 628 072	·007246377	13988
139	... 1 93 21	... 2 685 619	.....·007194245	14301
140	1 96 00	2 744 000	·007142857	14613
141	1 98 81	2 803 221	·007092199	14922
142	... 2 01 64	... 2 863 288	.....·007042254	15229
143	2 04 49	2 924 207	·006993007	15534
144	2 07 36	2 985 984	·006944444	15836
145	2 10 25	3 048 625	·006896552	16137

No.	Square.	Cube.	Reciprocal.	C. Log.
146	2 13 16	3 112 136	·006849315	16435
147	2 16 09	3 176 523	·006802721	16732
148	... 2 19 04	... 3 241 792	.....·006756757	17026
149	2 22 01	3 307 949	·006711409	17319
150	2 25 00	3 375 000	·006666667	17609
151	... 2 28 01	... 3 442 951	.....·006622517	17898
152	2 31 04	3 511 808	·006578947	18184
153	2 34 09	3 581 577	·006535948	18469
154	... 2 37 16	... 3 652 264	.....·006493506	18752
155	2 40 25	3 723 875	·006451613	19033
156	2 43 36	3 796 416	·006410256	19312
157	... 2 46 49	... 3 869 893	.....·006369427	19590
158	2 49 64	3 944 312	·006329114	19866
159	2 52 81	4 019 679	·006289308	20140
160	... 2 56 00	... 4 096 000	.....·006250000	20412
161	2 59 21	4 173 281	·006211180	20683
162	2 62 44	4 251 528	·006172840	20952
163	... 2 65 69	... 4 330 747	.....·006134969	21219
164	2 68 96	4 410 944	·006097561	21484
165	2 72 25	4 492 125	·006060606	21748
166	... 2 75 56	... 4 574 296	.....·006024096	22011
167	2 78 89	4 657 463	·005988024	22272
168	2 82 24	4 741 632	·005952381	22531
169	... 2 85 61	... 4 826 809	.....·005917160	22789
170	2 89 00	4 913 000	·005882353	23045
171	2 92 41	5 000 211	·005847953	23300
172	... 2 95 84	... 5 088 448	.....·005813953	23553
173	2 99 29	5 177 717	·005780347	23805
174	3 02 76	5 268 024	·005747126	24055
175	... 3 06 25	... 5 359 375	.....·005714286	24304
176	3 09 76	5 451 776	·005681818	24551
177	3 13 29	5 545 233	·005649718	24797
178	... 3 16 84	... 5 639 752	.....·005617978	25042
179	3 20 41	5 735 339	·005586592	25285
180	3 24 00	5 832 000	·005555556	25527
181	... 3 27 61	... 5 929 741	.....·005524862	25768
182	3 31 24	6 028 568	·005494505	26007
183	3 34 89	6 128 487	·005464481	26245
184	... 3 38 56	... 6 229 504	.....·005434783	26482
185	3 42 25	6 331 625	·005405405	26717
186	3 45 96	6 434 856	·005376344	26951
187	... 3 49 69	... 6 539 203	.....·005347594	27184
188	3 53 44	6 644 672	·005319149	27416
189	3 57 21	6 751 269	·005291005	27646
190	3 61 00	6 859 000	·00526315[illegible]	27875

No.	Square.	Cube.	Reciprocal.	C. Log.
191	3 64 81	6 967 871	·005235602	28103
192	3 68 64	7 077 888	·005208333	28330
193	... 3 72 49	... 7 189 057	.....·005181347	28556
194	3 76 36	7 301 384	·005154639	28780
195	3 80 25	7 414 875	·005128205	29003
196	... 3 84 16	... 7 529 536	.....·005102041	29226
197	3 88 09	7 645 373	·005076142	29447
198	3 92 04	7 762 392	·005050505	29667
199	... 3 96 01	... 7 880 599	.....·005025126	29885
200	4 00 00	8 000 000	·005000000	30103
201	4 04 01	8 120 601	·004975124	30320
202	... 4 08 04	... 8 242 408	.....·004950495	30535
203	4 12 09	8 365 427	·004926108	30750
204	4 16 16	8 489 664	·004901961	30963
205	... 4 20 25	... 8 615 125	.....·004878049	31175
206	4 24 36	8 741 816	·004854369	31387
207	4 28 49	8 869 743	·004830918	31597
208	... 4 32 64	... 8 998 912	.....·004807692	31806
209	4 36 81	9 129 329	·004784689	32015
210	4 41 00	9 261 000	·004761905	32222
211	... 4 45 21	... 9 393 931	.....·004739336	32428
212	4 49 44	9 528 128	·004716981	32634
213	4 53 69	9 663 597	·004694836	32838
214	... 4 57 96	... 9 800 344	.....·004672897	33041
215	4 62 25	9 938 375	·004651163	33244
216	4 66 56	10 077 696	·004629630	33445
217	... 4 70 89	...10 218 313	.....·004608295	33646
218	4 75 24	10 360 232	·004587156	33846
219	4 79 61	10 503 459	·004566210	34044
220	... 4 84 00	...10 648 000	.....·004545455	34242
221	4 88 41	10 793 861	·004524887	34439
222	4 92 84	10 941 048	·004504505	34635
223	... 4 97 29	...11 089 567	.....·004484305	34830
224	5 01 76	11 239 424	·004464286	35025
225	5 06 25	11 390 625	·004444444	35218
226	... 5 10 76	...11 543 176	.....·004424779	35411
227	5 15 29	11 697 083	·004405286	35603
228	5 19 84	11 852 352	·004385965	35793
229	... 5 24 41	...12 008 989	.....·00436681 2	35984
230	5 29 00	12 167 000	·004347826	36173
231	5 33 61	12 326 391	·004329004	36361
232	... 5 38 24	...12 487 168	.....·004310345	36549
233	5 42 89	12 649 337	·004291845	36736
234	5 47 56	12 812 904	·004273504	36922
235	5 52 25	12 977 875	·004255319	37107

No.	Square.	Cube.	Reciprocal.	C. Log.
236	5 56 96	13 144 256	·004237288	37291
237	5 61 69	13 312 053	·004219409	37475
238	... 5 66 44	...13 481 272	.....·004201681	37658
239	5 71 21	13 651 919	·004184100	37840
240	5 76 00	13 824 000	·004166667	38021
241	... 5 80 81	...13 997 521	.....·004149378	38202
242	5 85 64	14 172 488	·004132231	38382
243	5 90 49	14 348 907	·004115226	38561
244	... 5 95 36	...14 526 784	.....·004098361	38739
245	6 00 25	14 706 125	·004081633	38917
246	6 05 16	14 886 936	·004065041	39094
247	... 6 10 09	...15 069 223	.....·004048583	39270
248	6 15 04	15 252 992	·004032258	39445
249	6 20 01	15 438 249	·004016064	39620
250	... 6 25 00	...15 625 000	.....·004000000	39794
251	6 30 01	15 813 251	·003984064	39967
252	6 35 04	16 003 008	·003968254	40140
253	... 6 40 09	...16 194 277	.....·003952569	40312
254	6 45 16	16 387 064	·003937008	40483
255	6 50 25	16 581 375	·003921569	40654
256	... 6 55 36	...16 777 216	.....·003906250	40824
257	6 60 49	16 974 593	·003891051	40993
258	6 65 64	17 173 512	·003875969	41162
259	... 6 70 81	...17 373 979	.....·003861004	41330
260	6 76 00	17 576 000	·003846154	41497
261	6 81 21	17 779 581	·003831418	41664
262	... 6 86 44	...17 984 728	.....·003816794	41830
263	6 91 69	18 191 447	·003802281	41996
264	6 96 96	18 399 744	·003787879	42160
265	... 7 02 25	...18 609 625	.....·003773585	42325
266	7 07 56	18 821 096	·003759398	42488
267	7 12 89	19 034 163	·003745318	42651
268	... 7 18 24	...19 248 832	.....·003731343	42813
269	7 23 61	19 465 109	·003717472	42975
270	7 29 00	19 683 000	·003703704	43136
271	... 7 34 41	...19 902 511	.....·003690037	43297
272	7 39 84	20 123 648	·003676471	43457
273	7 45 29	20 346 417	·003663004	43616
274	... 7 50 76	...20 570 824	.....·003649635	43775
275	7 56 25	20 796 875	·003636364	43933
276	7 61 76	21 024 576	·003623188	44091
277	... 7 67 29	...21 253 933	.....·003610108	44248
278	7 72 84	21 484 952	·003597122	44404
279	7 78 41	21 717 639	·003584229	44560
280	7 84 00	21 952 000	·003571429	44716

No.	Square.	Cube.	Reciprocal.	C. Log.
281	7 89 61	22 188 041	·00355871 9	44671
282	7 95 24	22 425 768	·00354609 9	45025
283	... 8 00 89	...22 665 187	.....·00353356 9	45179
284	8 06 56	22 906 304	·00352112 7	45332
285	8 12 25	23 149 125	·00350877 2	45484
286	... 8 17 96	...23 393 656	.....·00349650 3	45637
287	8 23 69	23 639 903	·00348432 1	45788
288	8 29 44	23 887 872	·00347222 2	45939
289	... 8 35 21	...24 137 569	.....·00346020 8	46090
290	8 41 00	24 389 000	·00344827 6	46240
291	8 46 81	24 642 171	·00343642 6	46389
292	... 8 52 64	...24 897 088	.....·00342465 8	46538
293	8 58 49	25 153 757	·00341296 9	46687
294	8 64 36	25 412 184	·00340136 1	46835
295	... 8 70 25	...25 672 375	.....·00338983 1	46982
296	8 76 16	25 934 336	·00337837 8	47129
297	8 82 09	26 198 073	·00336700 3	47276
298	... 8 88 04	...26 463 592	.....·00335570 5	47422
299	8 94 01	26 730 899	·00334448 2	47567
300	9 00 00	27 000 000	·00333333 3	47712
301	... 9 06 01	...27 270 901	.....·00332225 9	47857
302	9 12 04	27 543 608	·00331125 8	48001
303	9 18 09	27 818 127	·00330033 0	48144
304	... 9 24 16	...28 094 464	.....·00328947 4	48287
305	9 30 25	28 372 625	·00327868 9	48430
306	9 36 36	28 652 616	·00326797 4	48572
307	... 9 42 49	...28 934 443	.....·00325732 9	48714
308	9 48 64	29 218 112	·00324675 3	48855
309	9 54 81	29 503 629	·00323624 6	48996
310	... 9 61 00	...29 791 000	.....·00322580 6	49136
311	9 67 21	30 080 231	·00321543 4	49276
312	9 73 44	30 371 328	·00320512 8	49415
313	... 9 79 69	...30 664 297	.....·00319488 8	49554
314	9 85 96	30 959 144	·00318471 3	49693
315	9 92 25	31 255 875	·00317460 3	49831
316	... 9 98 56	...31 554 496	.....·00316455 7	49969
317	10 04 89	31 855 013	·00315457 4	50106
318	10 11 24	32 157 432	·00314465 4	50243
319	... 10 17 61	...32 461 759	.....·00313479 6	50379
320	10 24 00	32 768 000	·00312500 0	50515
321	10 30 41	33 076 161	·00311526 5	50651
322	... 10 36 84	...33 386 248	.....·00310559 0	50786
323	10 43 29	33 698 267	·00309597 5	50920
324	10 49 76	34 012 224	·00308642 0	51055
325	10 56 25	34 328 125	·00307692 3	51188

No.	Square.	Cube.	Reciprocal.	C. Log.
326	10 62 76	34 645 976	·003067485	51322
327	10 69 29	34 965 783	·003058104	51455
328	... 10 75 84	...35 287 552	.....·003048780	51587
329	10 82 41	35 611 289	·003039514	51720
330	10 89 00	35 937 000	·003030303	51851
331	... 10 95 61	...36 264 691	.....·003021148	51983
332	11 02 24	36 594 368	·003012048	52114
333	11 08 89	36 926 037	·003003003	52244
334	... 11 15 56	...37 259 704	.....·002994012	52375
335	11 22 25	37 595 375	·002985075	52504
336	11 28 96	37 933 056	·002976190	52634
337	... 11 35 69	...38 272 753	.....·002967359	52763
338	11 42 44	38 614 472	·002958580	52892
339	11 49 21	38 958 219	·002949853	53020
340	... 11 56 00	...39 304 000	.....·002941176	53148
341	11 62 81	39 651 821	·002932551	53275
342	11 69 64	40 001 688	·002923977	53403
343	... 11 76 49	...40 353 607	.....·002915452	53529
344	11 83 36	40 707 584	·002906977	53656
345	11 90 25	41 063 625	·002898551	53782
346	... 11 97 16	...41 421 736	.....·002890173	53908
347	12 04 09	41 781 923	·002881844	54033
348	12 11 04	42 144 192	·002873563	54158
349	... 12 18 01	...42 508 549	.....·002865330	54283
350	12 25 00	42 875 000	·002857143	54407
351	12 32 01	43 243 551	·002849003	54531
352	... 12 39 04	...43 614 208	.....·002840909	54654
353	12 46 09	43 986 977	·002832861	54777
354	12 53 16	44 361 864	·002824859	54900
355	... 12 60 25	...44 738 875	.....·002816901	55023
356	12 67 36	45 118 016	·002808989	55145
357	12 74 49	45 499 293	·002801120	55267
358	... 12 81 64	...45 882 712	.....·002793296	55388
359	12 88 81	46 268 279	·002785515	55509
360	12 96 00	46 656 000	·002777778	55630
361	... 13 03 21	...47 045 881	.....·002770083	55751
362	13 10 44	47 437 928	·002762431	55871
363	13 17 69	47 832 147	·002754821	55991
364	... 13 24 96	...48 228 544	.....·002747253	56110
365	13 32 25	48 627 125	·002739726	56229
366	13 39 56	49 027 896	·002732240	56348
367	... 13 46 89	...49 430 863	.....·002724796	56467
368	13 54 24	49 836 032	·002717391	56585
369	13 61 61	50 243 409	·002710027	56703
370	13 69 00	50 653 000	·002702703	56820

No.	Square.	Cube.	Reciprocal.	C. Log.
371	13 76 41	51 064 811	·002695418	56937
372	13 83 84	51 478 848	·002688172	57054
373	... 13 91 29	...51 895 117	.....·002680965	57171
374	13 98 76	52 313 624	·002673797	57287
375	14 06 25	52 734 375	·002666667	57403
376	... 14 13 76	...53 157 376	.....·002659574	57519
377	14 21 29	53 582 633	·002652520	57634
378	14 28 84	54 010 152	·002645503	57749
379	... 14 36 41	...54 439 939	.....·002638522	57864
380	14 44 00	54 872 000	·002631579	57978
381	14 51 61	55 306 341	·002624672	58092
382	... 14 59 24	...55 742 968	.....·002617801	58206
383	14 66 89	56 181 887	·002610966	58320
384	14 74 56	56 623 104	·002604167	58433
385	... 14 82 25	...57 066 625	.....·002597403	58546
386	14 89 96	57 512 456	·002590674	58659
387	14 97 69	57 960 603	·002583979	58771
388	... 15 05 44	...58 411 072	.....·002577320	58883
389	15 13 21	58 863 869	·002570694	58995
390	15 21 00	59 319 000	·002564103	59106
391	... 15 28 81	...59 776 471	.....·002557545	59218
392	15 36 64	60 236 288	·002551020	59329
393	15 44 49	60 698 457	·002544529	59439
394	... 15 52 36	...61 162 984	.....·002538071	59550
395	15 60 25	61 629 875	·002531646	59660
396	15 68 16	62 099 136	·002525253	59770
397	... 15 76 09	...62 570 773	.....·002518892	59879
398	15 84 04	63 044 792	·002512563	59988
399	15 92 01	63 521 199	·002506266	60097
400	... 16 00 00	...64 000 000	.....·002500000	60206
401	16 08 01	64 481 201	·002493766	60314
402	16 16 04	64 964 808	·002487562	60423
403	... 16 24 09	...65 450 827	.....·002481390	60531
404	16 32 16	65 939 264	·002475248	60638
405	16 40 25	66 430 125	·002469136	60746
406	... 16 48 36	...66 923 416	.....·002463054	60853
407	16 56 49	67 419 143	·002457002	60959
408	16 64 64	67 917 312	·002450980	61066
409	... 16 72 81	...68 417 929	.....·002444988	61172
410	16 81 00	68 921 000	·002439024	61278
411	16 89 21	69 426 531	·002433090	61384
412	... 16 97 44	...69 934 528	.....·002427184	61490
413	17 05 69	70 444 997	·002421308	61595
414	17 13 96	70 957 944	·002415459	61700
415	17 22 25	71 473 375	·002409639	61805

No.	Square.	Cube.	Reciprocal.	C. Log.
416	17 30 56	71 991 296	·002403846	61909
417	17 38 89	72 511 713	·002398082	62014
418	... 17 47 24	...73 034 632	.....·002392344	62118
419	17 55 61	73 560 059	·002386635	62221
420	17 64 00	74 088 000	·002380952	62325
421	... 17 72 41	...74 618 461	.....·002375297	62428
422	17 80 84	75 151 448	·002369668	62531
423	17 89 29	75 686 967	·002364066	62634
424	... 17 97 76	...76 225 024	.....·002358491	62737
425	18 06 25	76 765 625	·002352941	62839
426	18 14 76	77 308 776	·002347418	62941
427	... 18 23 29	...77 854 483	.....·002341920	63043
428	18 31 84	78 402 752	·002336449	63144
429	18 40 41	78 953 589	·002331002	63246
430	... 18 49 00	...79 507 000	.....·002325581	63347
431	18 57 61	80 062 991	·002320186	63448
432	18 66 24	80 621 568	·002314815	63548
433	... 18 74 89	...81 182 737	.....·002309469	63649
434	18 83 56	81 746 504	·002304147	63749
435	18 92 25	82 312 875	·002298851	63849
436	... 19 00 96	...82 881 856	.....·002293578	63949
437	19 09 69	83 453 453	·002288330	64048
438	19 18 44	84 027 672	·002283105	64147
439	... 19 27 21	...84 604 519	.....·002277904	64246
440	19 36 00	85 184 000	·002272727	64345
441	19 44 81	85 766 121	·002267574	64444
442	... 19 53 64	...86 350 888	.....·002262443	64542
443	19 62 49	86 938 307	·002257336	64640
444	19 71 36	87 528 384	·002252252	64738
445	... 19 80 25	...88 121 125	.....·002247191	64836
446	19 89 16	88 716 536	·002242152	64933
447	19 98 09	89 314 623	·002237136	65031
448	... 20 07 04	...89 915 392	.....·002232143	65128
449	20 16 01	90 518 849	·002227171	65225
450	20 25 00	91 125 000	·002222222	65321
451	... 20 34 01	...91 733 851	.....·002217295	65418
452	20 43 04	92 345 408	·002212389	65514
453	20 52 09	92 959 677	·002207506	65610
454	... 20 61 16	...93 576 664	.....·002202643	65706
455	20 70 25	94 196 375	·002197802	65801
456	20 79 36	94 818 816	·002192982	65896
457	... 20 88 49	...95 443 993	.....·002188184	65992
458	20 97 64	96 071 912	·002183406	66087
459	21 06 81	96 702 579	·002178649	66181
460	21 16 00	97 336 000	·002173913	66276

No.	Square.	Cube.	Reciprocal.	C. Log.
461	21 25 21	97 972 181	·002169197	66370
462	21 34 44	98 611 128	·002164502	66464
463	... 21 43 69	... 99 252 847	.....·002159827	66558
464	21 52 96	99 897 344	·002155172	66652
465	21 62 25	100 544 625	·002150538	66745
466	... 21 71 56	...101 194 696	.....·002145923	66839
467	21 80 89	101 847 563	·002141328	66932
468	21 90 24	102 503 232	·002136752	67025
469	... 21 99 61	...103 161 709	.....·002132196	67117
470	22 09 00	103 823 000	·002127660	67210
471	22 18 41	104 487 111	·002123142	67302
472	... 22 27 84	...105 154 048	.....·002118644	67394
473	22 37 29	105 823 817	·002114165	67486
474	22 46 76	106 496 424	·002109705	67578
475	... 22 56 25	...107 171 875	.....·002105263	67669
476	22 65 76	107 850 176	·002100840	67761
477	22 75 29	108 531 333	·002096436	67852
478	... 22 84 84	...109 215 352	.....·002092050	67943
479	22 94 41	109 902 239	·002087683	68034
480	23 04 00	110 592 000	·002083333	68124
481	... 23 13 61	...111 284 641	.....·002079002	68215
482	23 23 24	111 980 168	·002074689	68305
483	23 32 89	112 678 587	·002070393	68395
484	... 23 42 56	...113 379 904	.....·002066116	68485
485	23 52 25	114 084 125	·002061856	68574
486	23 61 96	114 791 256	·002057613	68664
487	... 23 71 69	...115 501 303	.....·002053388	68753
488	23 81 44	116 214 272	·002049180	68842
489	23 91 21	116 930 169	·002044990	68931
490	... 24 01 00	...117 649 000	.....·002040816	69020
491	24 10 81	118 370 771	·002036660	69108
492	24 20 64	119 095 488	·002032520	69197
493	... 24 30 49	...119 823 157	.....·002028398	69285
494	24 40 36	120 553 784	·002024291	69373
495	24 50 25	121 287 375	·002020202	69461
496	... 24 60 16	...122 023 936	.....·002016129	69548
497	24 70 09	122 763 473	·002012072	69636
498	24 80 04	123 505 992	·002008032	69723
499	... 24 90 01	...124 251 499	.....·002004008	69810
500	25 00 00	125 000 000	·002000000	69897
501	25 10 01	125 751 501	·001996008	69984
502	... 25 20 04	...126 506 008	.....·001992032	70070
503	25 30 09	127 263 527	·001988072	70157
504	25 40 16	128 024 064	·001984127	70243
505	25 50 25	128 787 625	·001980198	70329

No.	Square.	Cube.	Reciprocal.	C. Log.
506	25 60 36	129 554 216	·001976285	70415
507	25 70 49	130 323 843	·001972387	70501
508	... 25 80 64	...131 096 512	.....·001968504	70586
509	25 90 81	131 872 229	·001964637	70672
510	26 01 00	132 651 000	·001960784	70757
511	... 26 11 21	...133 432 831	.....·001956947	70842
512	26 21 44	134 217 728	·001953125	70927
513	26 31 69	135 005 697	·001949318	71012
514	... 26 41 96	...135 796 744	.....·001945525	71096
515	26 52 25	136 590 875	·001941748	71181
516	26 62 56	137 388 096	·001937984	71265
517	... 26 72 89	...138 188 413	.....·001934236	71349
518	26 83 24	138 991 832	·001930502	71433
519	26 93 61	139 798 359	·001926782	71517
520	... 27 04 00	...140 608 000	.....·001923077	71600
521	27 14 41	141 420 761	·001919386	71684
522	27 24 84	142 236 648	·001915709	71767
523	... 27 35 29	...143 055 667	.....·001912046	71850
524	27 45 76	143 877 824	·001908397	71933
525	27 56 25	144 703 125	·001904762	72016
526	... 27 66 76	...145 531 576	.....·001901141	72099
527	27 77 29	146 363 183	·001897533	72181
528	27 87 84	147 197 952	·001893939	72263
529	... 27 98 41	...148 035 889	.....·001890359	72346
530	28 09 00	148 877 000	·001886792	72428
531	28 19 61	149 721 291	·001883239	72509
532	... 28 30 24	...150 568 768	.....·001879699	72591
533	28 40 89	151 419 437	·001876173	72673
534	28 51 56	152 273 304	·001872659	72754
535	... 28 62 25	...153 130 375	.....·001869159	72835
536	28 72 96	153 990 656	·001865672	72916
537	28 83 69	154 854 153	·001862197	72997
538	... 28 94 44	...155 720 872	.....·001858736	73078
539	29 05 21	156 590 819	·001855288	73159
540	29 16 00	157 464 000	·001851852	73239
541	... 29 26 81	...158 340 421	.....·001848429	73320
542	29 37 64	159 220 088	·001845018	73400
543	29 48 49	160 103 007	·001841621	73480
544	... 29 59 36	...160 989 184	.....·001838235	73560
545	29 70 25	161 878 625	·001834862	73640
546	29 81 16	162 771 336	·001831502	73719
547	... 29 92 09	...163 667 323	.....·001828154	73799
548	30 03 04	164 566 592	·001824818	73878
549	30 14 01	165 469 149	·001821494	73957
550	30 25 00	166 375 000	·001818182	74036

No.	Square.	Cube.	Reciprocal.	C. Log.
551	30 36 01	167 284 151	·001814882	74115
552	30 47 04	168 196 608	·001811594	74194
553	... 30 58 09	...169 112 377	.....·001808318	74273
554	30 69 16	170 031 464	·001805054	74351
555	30 80 25	170 953 875	·001801802	74429
556	... 30 91 36	...171 879 616	.....·001798561	74507
557	31 02 49	172 808 693	·001795332	74586
558	31 13 64	173 741 112	·001792115	74663
559	... 31 24 81	...174 676 879	.....·001788909	74741
560	31 36 00	175 616 000	·001785714	74819
561	31 47 21	176 558 481	·001782531	74896
562	... 31 58 44	...177 504 328	.....·001779359	74974
563	31 69 69	178 453 547	·001776199	75051
564	31 80 96	179 406 144	·001773050	75128
565	... 31 92 25	...180 362 125	.....·001769912	75205
566	32 03 56	181 321 496	·001766784	75282
567	32 14 89	182 284 263	·001763668	75358
568	... 32 26 24	...183 250 432	.....·001760563	75435
569	32 37 61	184 220 009	·001757469	75511
570	32 49 00	185 193 000	·001754386	75587
571	... 32 60 41	...186 169 411	.....·001751313	75664
572	32 71 84	187 149 248	·001748252	75740
573	32 83 29	188 132 517	·001745201	75815
574	... 32 94 76	...189 119 224	.....·001742160	75891
575	33 06 25	190 109 375	·001739130	75967
576	33 17 76	191 102 976	·001736111	76042
577	... 33 29 29	...192 100 033	.....·001733102	76118
578	33 40 84	193 100 552	·001730104	76193
579	33 52 41	194 104 539	·001727116	76268
580	... 33 64 00	...195 112 000	.....·001724138	76343
581	33 75 61	196 122 941	·001721170	76418
582	33 87 24	197 137 368	·001718213	76492
583	... 33 98 89	...198 155 287	.....·001715266	76567
584	34 10 56	199 176 704	·001712329	76641
585	34 22 25	200 201 625	·001709402	76716
586	... 34 33 96	...201 230 056	.....·001706485	76790
587	34 45 69	202 262 003	·001703578	76864
588	34 57 44	203 297 472	·001700680	76938
589	... 34 69 21	...204 336 469	.....·001697793	77012
590	34 81 00	205 379 000	·001694915	77085
591	34 92 81	206 425 071	·001692047	77159
592	... 35 04 64	...207 474 688	.....·001689189	77232
593	35 16 49	208 527 857	·001686341	77305
594	35 28 36	209 584 584	·001683502	77379
595	35 40 25	210 644 875	·001680672	77452

No.	Square.	Cube.	Reciprocal.	C. Log.
596	35 52 16	211 708 736	·001677852	77525
597	35 64 09	212 776 173	·001675042	77597
598	... 35 76 04	...213 847 192	.....·001672241	77670
599	35 88 01	214 921 799	·001669449	77743
600	36 00 00	216 000 000	·001666667	77815
601	... 36 12 01	...217 081 801	.....·001663894	77887
602	36 24 04	218 167 208	·001661130	77960
603	36 36 09	219 256 227	·001658375	78032
604	... 36 48 16	...220 348 864	.....·001655629	78104
605	36 60 25	221 445 125	·001652893	78176
606	36 72 36	222 545 016	·001650165	78247
607	... 36 84 49	...223 648 543	.....·001647446	78319
608	36 96 64	224 755 712	·001644737	78390
609	37 08 81	225 866 529	·001642036	78462
610	... 37 21 00	...226 981 000	.....·001639344	78533
611	37 33 21	228 099 131	·001636661	78604
612	37 45 44	229 220 928	·001633987	78675
613	... 37 57 69	...230 346 397	.....·001631321	78746
614	37 69 96	231 475 544	·001628664	78817
615	37 82 25	232 608 375	·001626016	78888
616	... 37 94 56	...233 744 896	.....·001623377	78958
617	38 06 89	234 885 113	·001620746	79029
618	38 19 24	236 029 032	·001618123	79099
619	... 38 31 61	...237 176 659	.....·001615509	79169
620	38 44 00	238 328 000	·001612903	79239
621	38 56 41	239 483 061	·001610306	79309
622	... 38 68 84	...240 641 848	.....·001607717	79379
623	38 81 29	241 804 367	·001605136	79449
624	38 93 76	242 970 624	·001602564	79518
625	... 39 06 25	...244 140 625	.....·001600000	79588
626	39 18 76	245 314 376	·001597444	79657
627	39 31 29	246 491 883	·001594896	79727
628	... 39 43 84	...247 673 152	.....·001592357	79796
629	39 56 41	248 858 189	·001589825	79865
630	39 69 00	250 047 000	·001587302	79934
631	... 39 81 61	...251 239 591	.....·001584786	80003
632	39 94 24	252 435 968	·001582278	80072
633	40 06 89	253 636 137	·001579779	80140
634	... 40 19 56	...254 840 104	.....·001577287	80209
635	40 32 25	256 047 875	·001574803	80277
636	40 44 96	257 259 456	·001572327	80346
637	... 40 57 69	...258 474 853	.....·001569859	80414
638	40 70 44	259 694 072	·001567398	80482
639	40 83 21	260 917 119	·001564945	80550
640	40 96 00	262 144 000	·001562500	80618

No.	Square.	Cube.	Reciprocal.	C. Log.
641	41 08 81	263 374 721	·001560062	80686
642	41 21 64	264 609 288	·001557632	80754
643	... 41 34 49	...265 847 707	.....·001555210	80821
644	41 47 36	267 089 984	·001552795	80889
645	41 60 25	268 336 125	·001550388	80956
646	... 41 73 16	...269 586 136	.....·001547988	81023
647	41 86 09	270 840 023	·001545595	81090
648	41 99 04	272 097 792	·001543210	81158
649	... 42 12 01	...273 359 449	.....·001540832	81224
650	42 25 00	274 625 000	·001538462	81291
651	42 38 01	275 894 451	·001536098	81358
652	... 42 51 04	...277 167 808	.....·001533742	81425
653	42 64 09	278 445 077	·001531394	81491
654	42 77 16	279 726 264	·001529052	81558
655	... 42 90 25	...281 011 375	.....·001526718	81624
656	43 03 36	282 300 416	·001524390	81690
657	43 16 49	283 593 393	·001522070	81757
658	... 43 29 64	...284 890 312	.....·001519757	81823
659	43 42 81	286 191 179	·001517451	81889
660	43 56 00	287 496 000	·001515152	81954
661	... 43 69 21	...288 804 781	.....·001512859	82020
662	43 82 44	290 117 528	·001510574	82086
663	43 95 69	291 434 247	·001508296	82151
664	... 44 08 96	...292 754 944	.....·001506024	82217
665	44 22 25	294 079 625	·001503759	82282
666	44 35 56	295 408 296	·001501502	82347
667	... 44 48 89	...296 740 963	.....·001499250	82413
668	44 62 24	298 077 632	·001497006	82478
669	44 75 61	299 418 309	·001494768	82543
670	... 44 89 00	...300 763 000	.....·001492537	82607
671	45 02 41	302 111 711	·001490313	82672
672	45 15 84	303 464 448	·001488095	82737
673	... 45 29 29	...304 821 217	.....·001485884	82802
674	45 42 76	306 182 024	·001483680	82866
675	45 56 25	307 546 875	·001481481	82930
676	... 45 69 76	...308 915 776	.....·001479290	82995
677	45 83 29	310 288 733	·001477105	83059
678	45 96 84	311 665 752	·001474926	83123
679	... 46 10 41	...313 046 839	.....·001472754	83187
680	46 24 00	314 432 000	·001470588	83251
681	46 37 61	315 821 241	·001468429	83315
682	... 46 51 24	...317 214 568	.....·001466276	83378
683	46 64 89	318 611 987	·001464129	83442
684	46 78 56	320 013 504	·001461988	83506
685	46 92 25	321 419 125	·00145985[illegible]	83569

No.	Square.	Cube.	Reciprocal.	C. Log.
686	47 05 96	322 828 856	·00145772 6	83632
687	47 19 69	324 242 703	·00145560 4	83696
688	... 47 33 44	...325 660 672	.....·00145348 8	83759
689	47 47 21	327 082 769	·00145137 9	83822
690	47 61 00	328 509 000	·00144927 5	83885
691	... 47 74 81	...329 939 371	.....·00144717 8	83948
692	47 88 64	331 373 888	·00144508 7	84011
693	48 02 49	332 812 557	·00144300 1	84073
694	... 48 16 36	...334 255 384	.....·00144092 2	84136
695	48 30 25	335 702 375	·00143884 9	84198
696	48 44 16	337 153 536	·00143678 2	84261
697	... 48 58 09	...338 608 873	.....·00143472 0	84323
698	48 72 04	340 068 392	·00143266 5	84386
699	48 86 01	341 532 099	·00143061 5	84448
700	... 49 00 00	...343 000 000	.....·00142857 1	84510
701	49 14 01	344 472 101	·00142653 4	84572
702	49 28 04	345 948 408	·00142450 1	84634
703	... 49 42 09	...347 428 927	.....·00142247 5	84696
704	49 56 16	348 913 664	·00142045 5	84757
705	49 70 25	350 402 625	·00141844 0	84819
706	... 49 84 36	...351 895 816	.....·00141643 1	84880
707	49 98 49	353 393 243	·00141442 7	84942
708	50 12 64	354 894 912	·00141242 9	85003
709	... 50 26 81	...356 400 829	.....·00141043 7	85065
710	50 41 00	357 911 000	·00140845 1	85126
711	50 55 21	359 425 431	·00140647 0	85187
712	... 50 69 44	...360 944 128	.....·00140449 4	85248
713	50 83 69	362 467 097	·00140252 5	85309
714	50 97 96	363 994 344	·00140056 0	85370
715	... 51 12 25	...365 525 875	.....·00139860 1	85431
716	51 26 56	367 061 696	·00139664 8	85491
717	51 40 89	368 601 813	·00139470 0	85552
718	... 51 55 24	...370 146 232	.....·00139275 8	85612
719	51 69 61	371 694 959	·00139082 1	85673
720	51 84 00	373 248 000	·00138888 9	85733
721	... 51 98 41	...374 805 361	.....·00138696 3	85794
722	52 12 84	376 367 048	·00138504 2	85854
723	52 27 29	377 933 067	·00138312 6	85914
724	... 52 41 76	...379 503 424	.....·00138121 5	85974
725	52 56 25	381 078 125	·00137931 0	86034
726	52 70 76	382 657 176	·00137741 0	86094
727	... 52 85 29	...384 240 583	.....·00137551 6	86153
728	52 99 84	385 828 352	·00137362 6	86213
729	53 14 41	387 420 489	·00137174 2	86273
730	53 29 00	389 017 000	·00136986 3	86332

No.	Square.	Cube.	Reciprocal.	C. Log.
731	53 43 61	390 617 891	·001367989	86392
732	53 58 24	392 223 168	·001366120	86451
733	... 53 72 89	...393 832 837	.....·001364256	86510
734	53 87 56	395 446 904	·001362398	86570
735	54 02 25	397 065 375	·001360544	86629
736	... 54 16 96	...398 688 256	.....·001358696	86688
737	54 31 69	400 315 553	·001356852	86747
738	54 46 44	401 947 272	·001355014	86806
739	... 54 61 21	...403 583 419	.....·001353180	86864
740	54 76 00	405 224 000	·001351351	86923
741	54 90 81	406 869 021	·001349528	86982
742	... 55 05 64	...408 518 488	.....·001347709	87040
743	55 20 49	410 172 407	·001345895	87099
744	55 35 36	411 830 784	·001344086	87157
745	... 55 50 25	...413 493 625	.....·001342282	87216
746	55 65 16	415 160 936	·001340483	87274
747	55 80 09	416 832 723	·001338688	87332
748	... 55 95 04	...418 508 992	.....·001336898	87390
749	56 10 01	420 189 749	·001335113	87448
750	56 25 00	421 875 000	·001333333	87506
751	... 56 40 01	...423 564 751	.....·001331558	87564
752	56 55 04	425 259 008	·001329787	87622
753	56 70 09	426 957 777	·001328021	87679
754	... 56 85 16	...428 661 064	.....·001326260	87737
755	57 00 25	430 368 875	·001324503	87795
756	57 15 36	432 081 216	·001322751	87852
757	... 57 30 49	...433 798 093	.....·001321004	87910
758	57 45 64	435 519 512	·001319261	87967
759	57 60 81	437 245 479	·001317523	88024
760	... 57 76 00	...438 976 000	.....·001315789	88081
761	57 91 21	440 711 081	·001314060	88138
762	58 06 44	442 450 728	·001312336	88195
763	... 58 21 69	...444 194 947	.....·001310616	88252
764	58 36 96	445 943 744	·001308901	88309
765	58 52 25	447 697 125	·001307190	88366
766	... 58 67 56	...449 455 096	.....·001305483	88423
767	58 82 89	451 217 663	·001303781	88480
768	58 98 24	452 984 832	·001302083	88536
769	... 59 13 61	...454 756 609	.....·001300390	88593
770	59 29 00	456 533 000	·001298701	88649
771	59 44 41	458 314 011	·001297017	88705
772	... 59 59 84	...460 099 648	.....·001295337	88762
773	59 75 29	461 889 917	·001293661	88818
774	59 90 76	463 684 824	·001291990	88874
775	60 06 25	465 484 375	·001290323	88930

No.	Square.	Cube.	Reciprocal.	C. Log.
776	60 21 76	467 288 576	·001288660	88986
777	60 37 29	469 097 433	·001287001	89042
778	... 60 52 84	...470 910 952	.....·001285347	89098
779	60 68 41	472 729 139	·001283697	89154
780	60 84 00	474 552 000	·001282051	89209
781	... 60 99 61	...476 379 541	.....·001280410	89265
782	61 15 24	478 211 768	·001278772	89321
783	61 30 89	480 048 687	·001277139	89376
784	... 61 46 56	...481 890 304	.....·001275510	89432
785	61 62 25	483 736 625	·001273885	89487
786	61 77 96	485 587 656	·001272265	89542
787	... 61 93 69	...487 443 403	.....·001270648	89597
788	62 09 44	489 303 872	·001269036	89653
789	62 25 21	491 169 069	·001267427	89708
790	... 62 41 00	...493 039 000	.....·001265823	89763
791	62 56 81	494 913 671	·001264223	89818
792	62 72 64	496 793 088	·001262626	89873
793	... 62 88 49	...498 677 257	.....·001261034	89927
794	63 04 36	500 566 184	·001259446	89982
795	63 20 25	502 459 875	·001257862	90037
796	... 63 36 16	...504 358 336	.....·001256281	90091
797	63 52 09	506 261 573	·001254705	90146
798	63 68 04	508 169 592	·001253133	90200
799	... 63 84 01	...510 082 399	.....·001251564	90255
800	64 00 00	512 000 000	·001250000	90309
801	64 16 01	513 922 401	·001248439	90363
802	... 64 32 04	...515 849 608	.....·001246883	90417
803	64 48 09	517 781 627	·001245330	90472
804	64 64 16	519 718 464	·001243781	90526
805	... 64 80 25	...521 660 125	.....·001242236	90580
806	64 96 36	523 606 616	·001240695	90634
807	65 12 49	525 557 943	·001239157	90687
808	... 65 28 64	...527 514 112	.....·001237624	90741
809	65 44 81	529 475 129	·001236094	90795
810	65 61 00	531 441 000	·001234568	90849
811	... 65 77 21	...533 411 731	.....·001233046	90902
812	65 93 44	535 387 328	·001231527	90956
813	66 09 69	537 367 797	·001230012	91009
814	... 66 25 96	...539 353 144	.....·001228501	91062
815	66 42 25	541 343 375	·001226994	91116
816	66 58 56	543 338 496	·001225490	91169
817	... 66 74 89	...545 338 513	.....·001223990	91222
818	66 91 24	547 343 432	·001222494	91275
819	67 07 61	549 353 259	·001221001	91328
820	67 24 00	551 368 000	·001219512	91381

No.	Square.	Cube.	Reciprocal.	C. Log.
821	67 40 41	553 387 661	·001218027	91434
822	67 56 84	555 412 248	·001216545	91487
823	... 67 73 29	...557 441 767	.....·001215067	91540
824	67 89 76	559 476 224	·001213592	91593
825	68 06 25	561 515 625	·001212121	91645
826	... 68 22 76	...563 559 976	.....·001210654	91698
827	68 39 29	565 609 283	·001209190	91751
828	68 55 84	567 663 552	·001207729	91803
829	... 68 72 41	...569 722 789	.....·001206273	91855
830	68 89 00	571 787 000	·001204819	91908
831	69 05 61	573 856 191	·001203369	91960
832	... 69 22 24	...575 930 368	.....·001201923	92012
833	69 38 89	578 009 537	·001200480	92065
834	69 55 56	580 093 704	·001199041	92117
835	... 69 72 25	...582 182 875	.....·001197605	92169
836	69 88 96	584 277 056	·001196172	92221
837	70 05 69	586 376 253	·001194743	92273
838	... 70 22 44	...588 480 472	.....·001193317	92324
839	70 39 21	590 589 719	·001191895	92376
840	70 56 00	592 704 000	·001190476	92428
841	... 70 72 81	...594 823 321	.....·001189061	92480
842	70 89 64	596 947 688	·001187648	92531
843	71 06 49	599 077 107	·001186240	92583
844	... 71 23 36	...601 211 584	.....·001184834	92634
845	71 40 25	603 351 125	·001183432	92686
846	71 57 16	605 495 736	·001182033	92737
847	... 71 74 09	...607 645 423	.....·001180638	92788
848	71 91 04	609 800 192	·001179245	92840
849	72 08 01	611 960 049	·001177856	92891
850	... 72 25 00	...614 125 000	.....·001176471	92942
851	72 42 01	616 295 051	·001175088	92993
852	72 59 04	618 470 208	·001173709	93044
853	... 72 76 09	...620 650 477	.....·001172333	93095
854	72 93 16	622 835 864	·001170960	93146
855	73 10 25	625 026 375	·001169591	93197
856	... 73 27 36	...627 222 016	.....·001168224	93247
857	73 44 49	629 422 793	·001166861	93298
858	73 61 64	631 628 712	·001165501	93349
859	... 73 78 81	...633 839 779	.....·001164144	93399
860	73 96 00	636 056 000	·001162791	93450
861	74 13 21	638 277 381	·001161440	93500
862	... 74 30 44	...640 503 928	.....·001160093	93551
863	74 47 69	642 735 647	·001158749	93601
864	74 64 96	644 972 544	·001157407	93651
865	74 82 25	647 214 625	·001156069	93702

No.	Square.	Cube.	Reciprocal.	C. Log.
866	74 99 56	649 461 896	·00115473 4	93752
867	75 16 89	651 714 363	·00115340 3	93802
868	... 75 34 24	...653 972 032	.....·00115207 4	93852
869	75 51 61	656 234 909	·00115074 8	93902
870	75 69 00	658 503 000	·00114942 5	93952
871	... 75 86 41	...660 776 311	.....·00114810 6	94002
872	76 03 84	663 054 848	·00114678 9	94052
873	76 21 29	665 338 617	·00114547 5	94101
874	... 76 38 76	...667 627 624	.....·00114416 5	94151
875	76 56 25	669 921 875	·00114285 7	94201
876	76 73 76	672 221 376	·00114155 3	94250
877	... 76 91 29	...674 526 133	.....·00114025 1	94300
878	77 08 84	676 836 152	·00113895 2	94349
879	77 26 41	679 151 439	·00113765 6	94399
880	... 77 44 00	...681 472 000	.....·00113636 4	94448
881	77 61 61	683 797 841	·00113507 4	94498
882	77 79 24	686 128 968	·00113378 7	94547
883	... 77 96 89	...688 465 387	.....·00113250 3	94596
884	78 14 56	690 807 104	·00113122 2	94645
885	78 32 25	693 154 125	·00112994 4	94694
886	... 78 49 96	...695 506 456	.....·00112866 8	94743
887	78 67 69	697 864 103	·00112739 6	94792
888	78 85 44	700 227 072	·00112612 6	94841
889	... 79 03 21	...702 595 369	.....·00112485 9	94890
890	79 21 00	704 969 000	·00112359 6	94939
891	79 38 81	707 347 971	·00112233 4	94988
892	... 79 56 64	...709 732 288	.....·00112107 6	95036
893	79 74 49	712 121 957	·00111982 1	95085
894	79 92 36	714 516 984	·00111856 8	95134
895	... 80 10 25	...716 917 375	.....·00111731 8	95182
896	80 28 16	719 323 136	·00111606 1	95231
897	80 46 09	721 734 273	·00111481 7	95279
898	... 80 64 04	...724 150 792	.....·00111358 6	95328
899	80 82 01	726 572 699	·00111234 7	95376
900	81 00 00	729 000 000	·00111111 1	95424
901	... 81 18 01	...731 432 701	.....·00110987 8	95472
902	81 36 04	733 870 808	·00110864 7	95521
903	81 54 09	736 314 327	·00110742 0	95569
904	... 81 72 16	...738 763 264	.....·00110619 5	95617
905	81 90 25	741 217 625	·00110497 2	95665
906	82 08 36	743 677 416	·00110375 3	95713
907	... 82 26 49	...746 142 643	.....·00110253 6	95761
908	82 44 64	748 613 312	·00110132 2	95809
909	82 62 81	751 089 429	·00110011 0	95856
910	82 81 00	753 571 000	·00109890 1	95904

No.	Square.	Cube.	Reciprocal.	C. Log.
911	82 99 21	756 058 031	·001097695	95952
912	83 17 44	758 550 528	·001096491	95999
913	... 83 35 69	...761 048 497	.....·001095290	96047
914	83 53 96	763 551 944	·001094092	96095
915	83 72 25	766 060 875	·001092896	96142
916	... 83 90 56	...768 575 296	.....·001091703	96190
917	84 08 89	771 095 213	·001090513	96237
918	84 27 24	773 620 632	·001089325	96284
919	... 84 45 61	...776 151 559	.....·001088139	96332
920	84 64 00	778 688 000	·001086957	96379
921	84 82 41	781 229 961	·001085776	96426
922	... 85 00 84	...783 777 448	.....·001084599	96473
923	85 19 29	786 330 467	·001083424	96520
924	85 37 76	788 889 024	·001082251	96567
925	... 85 56 25	...791 453 125	.....·001081081	96614
926	85 74 76	794 022 776	·001079914	96661
927	85 93 29	796 597 983	·001078749	96708
928	... 86 11 84	...799 178 752	.....·001077586	96755
929	86 30 41	801 765 089	·001076426	96802
930	86 49 00	804 357 000	·001075269	96848
931	... 86 67 61	...806 954 491	.....·001074114	96895
932	86 86 24	809 557 568	·001072961	96942
933	87 04 89	812 166 237	·001071811	96988
934	... 87 23 56	...814 780 504	.....·001070664	97035
935	87 42 25	817 400 375	·001069519	97081
936	87 60 96	820 025 856	·001068376	97128
937	... 87 79 69	...822 656 953	.....·001067236	97174
938	87 98 44	825 293 672	·001066098	97220
939	88 17 21	827 936 019	·001064963	97267
940	... 88 36 00	...830 584 000	.....·001063830	97313
941	88 54 81	833 237 621	·001062699	97359
942	88 73 64	835 896 888	·001061571	97405
943	... 88 92 49	...838 561 807	.....·001060445	97451
944	89 11 36	841 232 384	·001059322	97497
945	89 30 25	843 908 625	·001058201	97543
946	... 89 49 16	...846 590 536	.....·001057082	97589
947	89 68 09	849 278 123	·001055966	97635
948	89 87 04	851 971 392	·001054852	97681
949	... 90 06 01	...854 670 349	.....·001053741	97727
950	90 25 00	857 375 000	·001052632	97772
951	90 44 01	860 085 351	·001051525	97818
952	... 90 63 04	...862 801 408	.....·001050420	97864
953	90 82 09	865 523 177	·001049318	97909
954	91 01 16	868 250 664	·001048218	97955
955	91 20 25	870 983 875	·001047120	98000

No.	Square.	Cube.	Reciprocal.	C. Log.
956	91 39 36	873 722 816	·001046025	98046
957	91 58 49	876 467 493	·001044932	98091
958	... 91 77 64	...879 217 912	.....·001043841	98137
959	91 96 81	881 974 079	·001042753	98182
960	92 16 00	884 736 000	·001041667	98227
961	... 92 35 21	...887 503 681	.....·001040583	98272
962	92 54 44	890 277 128	·001039501	98318
963	92 73 69	893 056 347	·001038422	98363
964	... 92 92 96	...895 841 344	.....·001037344	98408
965	93 12 25	898 632 125	·001036269	98453
966	93 31 56	901 428 696	·001035197	98498
967	... 93 90 89	...904 231 063	.....·001034126	98543
968	93 70 24	907 039 232	·001033058	98588
969	93 89 61	909 853 209	·001031992	98632
970	... 94 09 00	...912 673 000	.....·001030928	98677
971	94 28 41	915 498 611	·001029866	98722
972	94 47 84	918 330 048	·001028807	98767
973	... 94 67 29	...921 167 317	.....·001027749	98811
974	94 86 76	924 010 424	·001026694	98856
975	95 06 25	926 859 375	·001025641	98900
976	... 95 25 76	...929 714 176	.....·001024590	98945
977	95 45 29	932 574 833	·001023541	98989
978	95 64 84	935 441 352	·001022495	99034
979	... 95 84 41	...938 313 739	.....·001021450	99078
980	96 04 00	941 192 000	·001020408	99123
981	96 23 61	944 076 141	·001019368	99167
982	... 96 43 24	...946 966 168	.....·001018330	99211
983	96 62 89	949 862 087	·001017294	99255
984	96 82 56	952 763 904	·001016260	99300
985	... 97 02 25	...955 671 625	.....·001015228	99344
986	97 21 96	958 585 256	·001014199	99388
987	97 41 69	961 504 803	·001013171	99432
988	... 97 61 44	...964 430 272	.....·001012146	99476
989	97 81 21	967 361 669	·001011122	99520
990	98 01 00	970 299 000	·001010101	99564
991	... 98 20 81	...973 242 271	.....·001009082	99607
992	98 40 64	976 191 488	·001008065	99651
993	98 60 49	979 146 657	·001007049	99695
994	... 98 80 36	...982 107 784	.....·001006036	99739
995	99 00 25	985 074 875	·001005025	99782
996	99 20 16	988 047 936	·001004016	99826
997	... 99 40 09	...991 026 973	.....·001003009	99870
998	99 60 04	994 011 992	·001002004	99913
999	99 80 01	997 002 999	·001001001	99957
1000	...100 00 00	...1000 000 000	.....·001000000	00000

Table of Circumferences and Areas of Circles.

Diam.	Circum.	Area.	Diam.	Circum.	Area.
101	317·30	8012	146	458·67	16742
102	320·44	8171	147	461·81	16972
103	323·58	8332	148	464·96	17203
104	326·73	8495	149	468·10	17437
105	329·87	8659	150	471·24	17671
106	333·01	8825	151	474·38	17908
107	336·15	8992	152	477·52	18146
108	339·29	9161	153	480·66	18385
109	342·43	9331	154	483·81	18627
110	345·58	9503	155	486·95	18869
111	348·72	9677	156	490·09	19113
112	351·86	9852	157	493·23	19359
113	355·00	10029	158	496·37	19607
114	358·14	10207	159	499·51	19856
115	361·28	10387	160	502·65	20106
116	364·42	10568	161	505·80	20358
117	367·57	10751	162	508·94	20612
118	370·71	10936	163	512·08	20867
119	373·85	11122	164	515·22	21124
120	376·99	11310	165	518·36	21382
121	380·13	11499	166	521·50	21642
122	383·27	11690	167	524·65	21904
123	386·42	11882	168	527·79	22167
124	389·56	12076	169	530·93	22432
125	392·70	12272	170	534·07	22698
126	395·84	12469	171	537·21	22966
127	398·98	12668	172	540·35	23235
128	402·12	12868	173	543·50	23506
129	405·27	13070	174	546·64	23779
130	408·41	13273	175	549·78	24053
131	411·55	13478	176	552·92	24329
132	414·69	13685	177	556·06	24606
133	417·83	13893	178	559·20	24885
134	420·97	14103	179	562·35	25165
135	424·12	14314	180	565·49	25447
136	427·26	14527	181	568·63	25730
137	430·40	14741	182	571·77	26016
138	433·54	14957	183	574·91	26302
139	436·68	15175	184	578·05	26590
140	439·82	15394	185	581·19	26880
141	442·96	15615	186	584·34	27172
142	446·11	15837	187	587·48	27465
143	449·25	16061	188	590·62	27759
144	452·39	16286	189	593·76	28055
145	455·53	16513	190	596·90	28353

Diam.	Circum.	Area.	Diam.	Circum.	Area.
191	600·04	28652	236	741·42	43744
192	603·19	28953	237	744·56	44115
193	606·33	29255	238	747·70	44488
194	609·47	29559	239	750·84	44863
195	612·61	29865	240	753·98	45239
196	615·75	30172	241	757·12	45617
197	618·89	30481	242	760·27	45996
198	622·04	30791	243	763·41	46377
199	625·18	31103	244	766·55	46759
200	628·32	31416	245	769·69	47144
201	631·46	31731	246	772·83	47529
202	634·60	32047	247	775·97	47916
203	637·74	32365	248	779·12	48305
204	640·89	32685	249	782·26	48695
205	644·03	33006	250	785·40	49087
206	647·17	33329	251	788·54	49481
207	650·31	33654	252	791·68	49876
208	653·45	33979	253	794·82	50273
209	656·59	34307	254	797·96	50671
210	659·73	34636	255	801·11	51071
211	662·88	34967	256	804·25	51472
212	666·02	35299	257	807·39	51875
213	669·16	35633	258	810·53	52279
214	672·30	35968	259	813·67	52685
215	675·44	36305	260	816·81	53093
216	678·58	36644	261	819·96	53502
217	681·73	36984	262	823·10	53913
218	684·87	37325	263	826·24	54325
219	688·01	37668	264	829·38	54739
220	691·15	38013	265	832·52	55155
221	694·29	38360	266	835·66	55572
222	697·43	38708	267	838·81	55990
223	700·58	39057	268	841·95	56410
224	703·72	39408	269	845·09	56832
225	706·86	39761	270	848·23	57256
226	710·00	40115	271	851·37	57680
227	713·14	40471	272	854·51	58107
228	716·28	40828	273	857·66	58535
229	719·42	41187	274	860·80	58965
230	722·57	41548	275	863·94	59396
231	725·71	41910	276	867·08	59828
232	728·85	42273	277	870·22	60263
233	731·99	42638	278	873·36	60699
234	735·13	43005	279	876·50	61136
235	738·27	43374	280	879·65	61575

Diam.	Circum.	Area.	D'am.	Circum.	Area.
281	882·79	62016	326	1024·16	83469
282	885·93	62458	327	1027·30	83982
283	889·07	62902	328	1030·44	84496
284	892·21	63347	329	1033·58	85012
285	895·35	63794	330	1036·73	85530
286	898·50	64242	331	1039·87	86049
287	901·64	64692	332	1043·01	86570
288	904·78	65144	333	1046·15	87092
289	907·92	65597	334	1049·29	87616
290	911·06	66052	335	1052·43	88141
291	914·20	66508	336	1055·58	88668
292	917·35	66966	337	1058·72	89197
293	920·49	67426	338	1061·86	89727
294	923·63	67887	339	1065·00	90259
295	926·77	68349	340	1068·14	90792
296	929·91	68813	341	1071·28	91327
297	933·05	69279	342	1074·42	91863
298	936·19	69747	343	1077·57	92401
299	939·34	70215	344	1080·71	92941
300	942·48	70686	345	1083·85	93482
301	945·62	71158	346	1086·99	94025
302	948·76	71631	347	1090·13	94569
303	951·90	72107	348	1093·27	95115
304	955·04	72583	349	1096·42	95662
305	958·19	73062	350	1099·56	96211
306	961·33	73542	351	1102·70	96762
307	964·47	74023	352	1105·84	97314
308	967·61	74506	353	1108·98	97868
309	970·75	74991	354	1112·12	98423
310	973·89	75477	355	1115·27	98980
311	977·04	75964	356	1118·41	99538
312	980·18	76454	357	1121·55	100098
313	983·32	76945	358	1124·69	100660
314	986·46	77437	359	1127·83	101223
315	989·60	77931	360	1130·97	101788
316	992·74	78427	361	1134·12	102354
317	995·88	78924	362	1137·26	102922
318	999·03	79423	363	1140·40	103491
319	1002·17	79923	364	1143·54	104062
320	1005·31	80425	365	1146·68	104635
321	1008·45	80928	366	1149·82	105209
322	1011·59	81433	367	1152·97	105785
323	1014·73	81940	368	1156·11	106362
324	1017·88	82448	369	1159·25	106941
325	1021·02	82958	370	1162·39	107521

Diam.	Circum.	Area.	Diam.	Circum.	Area.
371	1165·53	108103	416	1306·91	135918
372	1168·67	108687	417	1310·05	136572
373	1171·81	109272	418	1313·19	137228
374	1174·96	109858	419	1316·33	137885
375	1178·10	110447	420	1319·47	138544
376	1181·24	111036	421	1322·61	139205
377	1184·38	111628	422	1325·75	139867
378	1187·52	112221	423	1328·89	140531
379	1190·66	112815	424	1332·04	141196
380	1193·81	113411	425	1335·18	141863
381	1196·95	114009	426	1338·32	142531
382	1200·09	114608	427	1341·46	143201
383	1203·23	115209	428	1344·60	143872
384	1206·37	115812	429	1347·74	144545
385	1209·51	116416	430	1350·89	145220
386	1212·66	117021	431	1354·03	145896
387	1215·80	117628	432	1357·17	146574
388	1218·94	118237	433	1360·31	147254
389	1222·08	118847	434	1363·45	147934
390	1225·22	119459	435	1366·59	148617
391	1228·36	120072	436	1369·73	149301
392	1231·50	120687	437	1372·88	149987
393	1234·65	121304	438	1376·02	150674
394	1237·79	121922	439	1379·16	151363
395	1240·93	122542	440	1382·30	152053
396	1244·07	123163	441	1385·44	152745
397	1247·21	123786	442	1388·58	153439
398	1250·35	124410	443	1391·73	154134
399	1253·50	125036	444	1394·87	154830
400	1256·64	125664	445	1398·01	155528
401	1259·78	126293	446	1401·15	156228
402	1262·92	126923	447	1404·29	156930
403	1266·06	127556	448	1407·43	157633
404	1269·20	128190	449	1410·58	158337
405	1272·35	128825	450	1413·72	159043
406	1275·49	129462	451	1416·86	159751
407	1278·63	130100	452	1420·00	160460
408	1281·77	130741	453	1423·14	161171
409	1284·91	131382	454	1426·28	161883
410	1288·05	132025	455	1429·42	162597
411	1291·19	132670	456	1432·57	163313
412	1294·34	133317	457	1435·71	164030
413	1297·48	133965	458	1438·85	164748
414	1300·62	134614	459	1441·99	165468
415	1303·76	135265	460	1445·13	166190

Diam.	Circum.	Area.	Diam.	Circum.	Area.
461	1448·27	166914	506	1589·65	201090
462	1451·42	167639	507	1592·79	201886
463	1454·56	168365	508	1595·93	202683
464	1457·70	169093	509	1599·07	203482
465	1460·84	169823	510	1602·21	204282
466	1463·98	170554	511	1605·35	205084
467	1467·12	171287	512	1608·50	205887
468	1470·27	172021	513	1611·64	206692
469	1473·41	172757	514	1614·78	207499
470	1476·55	173494	515	1617·92	208307
471	1479·69	174234	516	1621·06	209117
472	1482·83	174974	517	1624·20	209928
473	1485·97	175716	518	1627·35	210741
474	1489·12	176460	519	1630·49	211556
475	1492·26	177205	520	1633·63	212372
476	1495·40	177952	521	1636·77	213189
477	1498·54	178701	522	1639·91	214008
478	1501·68	179451	523	1643·05	214829
479	1504·82	180203	524	1646·20	215651
480	1507·96	180956	525	1649·34	216475
481	1511·11	181711	526	1652·48	217301
482	1514·25	182467	527	1655·62	218128
483	1517·39	183225	528	1658·76	218956
484	1520·53	183984	529	1661·90	219787
485	1523·67	184745	530	1665·04	220618
486	1526·81	185508	531	1668·19	221452
487	1529·96	186272	532	1671·33	222287
488	1533·10	187038	533	1674·47	223123
489	1536·24	187805	534	1677·61	223961
490	1539·38	188574	535	1680·75	224801
491	1542·52	189345	536	1683·89	225642
492	1545·66	190117	537	1687·04	226484
493	1548·81	190890	538	1690·18	227329
494	1551·95	191665	539	1693·32	228175
495	1555·09	192442	540	1696·46	229022
496	1558·23	193221	541	1699·60	229871
497	1561·37	194000	542	1702·74	230722
498	1564·51	194782	543	1705·88	231574
499	1567·65	195565	544	1709·03	232428
500	1570·80	196350	545	1712·17	233283
501	1573·94	197136	546	1715·31	234140
502	1577·08	197923	547	1718·45	234998
503	1580·22	198713	548	1721·59	235858
504	1583·36	199504	549	1724·73	236720
505	1586·50	200296	550	1727·88	237583

Diam.	Circum.	Area.	Diam.	Circum.	Area.
551	1731·02	238448	596	1872·39	278986
552	1734·16	239314	597	1875·53	279923
553	1737·30	240182	598	1878·67	280862
554	1740·44	241051	599	1881·81	281802
555	1743·58	241922	600	1884·96	282743
556	1746·73	242795	601	1888·10	283687
557	1749·87	243669	602	1891·24	284631
558	1753·01	244545	603	1894·38	285578
559	1756·15	245422	604	1897·52	286526
560	1759·29	246301	605	1900·66	287475
561	1762·43	247181	606	1903·81	288426
562	1765·58	248063	607	1906·95	289379
563	1768·72	248947	608	1910·09	290334
564	1771·86	249832	609	1913·23	291289
565	1775·00	250719	610	1916·37	292247
566	1778·14	251607	611	1919·51	293206
567	1781·28	252497	612	1922·65	294166
568	1784·42	253388	613	1925·80	295128
569	1787·57	254281	614	1928·94	296092
570	1790·71	255176	615	1932·08	297057
571	1793·85	256072	616	1935·22	298024
572	1796·99	256970	617	1938·36	298992
573	1800·13	257869	618	1941·50	299962
574	1803·27	258770	619	1944·65	300934
575	1806·42	259672	620	1947·79	301907
576	1809·56	260576	621	1950·93	302882
577	1812·70	261482	622	1954·07	303858
578	1815·84	262389	623	1957·21	304836
579	1818·98	263298	624	1960·35	305815
580	1822·12	264208	625	1963·50	306796
581	1825·27	265120	626	1966·64	307779
582	1828·41	266033	627	1969·78	308763
583	1831·55	266948	628	1972·92	309748
584	1834·69	267865	629	1976·06	310736
585	1837·83	268783	630	1979·20	311725
586	1840·97	269702	631	1982·35	312715
587	1844·11	270624	632	1985·49	313707
588	1847·26	271547	633	1988·63	314700
589	1850·40	272471	634	1991·77	315696
590	1853·54	273397	635	1994·91	316692
591	1856·68	274325	636	1998·05	317690
592	1859·82	275254	637	2001·19	318690
593	1862·96	276184	638	2004·34	319692
594	1866·11	277117	639	2007·48	320695
595	1869·25	278052	640	2010·62	321699

Diam.	Circum.	Area.	Diam.	Circum.	Area.
641	2013·76	322705	686	2155·13	369605
642	2016·90	323713	687	2158·27	370684
643	2020·04	324722	688	2161·42	371764
644	2023·19	325733	689	2164·56	372845
645	2026·33	326745	690	2167·70	373928
646	2029·47	327759	691	2170·84	375013
647	2032·61	328775	692	2173·98	376099
648	2035·75	329792	693	2177·12	377187
649	2038·89	330810	694	2180·27	378276
650	2042·04	331831	695	2183·41	379367
651	2045·18	332853	696	2186·55	380459
652	2048·32	333876	697	2189·69	381554
653	2051·46	334901	698	2192·83	382649
654	2054·60	335927	699	2195·97	383746
655	2057·74	336955	700	2199·11	384845
656	2060·88	337985	701	2202·26	385945
657	2064·03	339016	702	2205·40	387047
658	2067·17	340049	703	2208·54	388151
659	2070·31	341084	704	2211·68	389256
660	2073·45	342119	705	2214·82	390363
661	2076·59	343157	706	2217·96	391471
662	2079·73	344196	707	2221·11	392580
663	2082·88	345237	708	2224·25	393692
664	2086·02	346279	709	2227·39	394805
665	2089·16	347323	710	2230·53	395919
666	2092·30	348368	711	2233·67	397035
667	2095·44	349415	712	2236·81	398153
668	2098·58	350464	713	2239·96	399272
669	2101·73	351514	714	2243·10	400393
670	2104·87	352565	715	2246·24	401515
671	2108·01	353618	716	2249·38	402639
672	2111·15	354673	717	2252·52	403765
673	2114·29	355730	718	2255·66	404892
674	2117·43	356788	719	2258·81	406020
675	2120·58	357847	720	2261·95	407150
676	2123·72	358908	721	2265·09	408282
677	2126·86	359971	722	2268·23	409416
678	2130·00	361035	723	2271·37	410550
679	2133·14	362101	724	2274·51	411687
680	2136·28	363168	725	2277·65	412825
681	2139·42	364237	726	2280·80	413965
682	2142·57	365308	727	2283·94	415106
683	2145·71	366380	728	2287·08	416248
684	2148·85	367453	729	2290·22	417393
685	2151·99	368528	730	2293·36	418539

Diam.	Circum.	Area.	Diam.	Circum.	Area.
731	2296·50	419686	776	2437·88	472948
732	2299·65	420835	777	2441·02	474168
733	2302·79	421986	778	2444·16	475389
734	2305·93	423139	779	2447·30	476612
735	2309·07	424293	780	2450·44	477836
736	2312·21	425448	781	2453·58	479062
737	2315·35	426604	782	2456·73	480290
738	2318·50	427762	783	2459·87	481519
739	2321·64	428922	784	2463·01	482750
740	2324·78	430084	785	2466·15	483982
741	2327·92	431247	786	2469·29	485216
742	2331·06	432412	787	2472·43	486451
743	2334·20	433578	788	2475·58	487688
744	2337·34	434746	789	2478·72	488927
745	2340·49	435916	790	2481·86	490167
746	2343·63	437087	791	2485·00	491409
747	2346·77	438259	792	2488·14	492652
748	2349·91	439433	793	2491·28	493897
749	2353·05	440609	794	2494·42	495143
750	2356·19	441786	795	2497·57	496391
751	2359·34	442965	796	2500·71	497641
752	2362·48	444146	797	2503·85	498892
753	2365·62	445328	798	2506·99	500145
754	2368·76	446511	799	2510·13	501399
755	2371·90	447697	800	2513·27	502655
756	2375·04	448883	801	2516·42	503912
757	2378·19	450072	802	2519·56	505171
758	2381·33	451262	803	2522·70	506432
759	2384·47	452453	804	2525·84	507694
760	2387·61	453646	805	2528·98	508958
761	2390·75	454841	806	2532·12	510223
762	2393·89	456037	807	2535·27	511490
763	2397·04	457234	808	2538·41	512758
764	2400·18	458434	809	2541·55	514028
765	2403·32	459635	810	2544·69	515300
766	2406·46	460837	811	2547·83	516573
767	2409·60	462041	812	2550·97	517848
768	2412·74	463247	813	2554·11	519124
769	2415·88	464454	814	2557·26	520402
770	2419·03	465663	815	2560·40	521681
771	2422·17	466873	816	2563·54	522962
772	2425·31	468085	817	2566·68	524245
773	2428·45	469298	818	2569·82	525529
774	2431·59	470513	819	2572·96	526814
775	2434·73	471730	820	2576·11	528102

Diam.	Circum.	Area.	Diam.	Circum.	Area.
821	2579·25	529391	866	2720·62	589014
822	2582·39	530681	867	2723·76	590375
823	2585·53	531973	868	2726·90	591738
824	2588·67	533267	869	2730·04	593102
825	2591·81	534562	870	2733·19	594468
826	2594·96	535858	871	2736·33	595835
827	2598·10	537157	872	2739·47	597204
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